

Phenomenal Distinctions and Observational Reconstruction

Descent, Definability, and Restricted Probes

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Abstract

We study reconstruction of specified distinctions from observations. We prove fiber and topos criteria, describe coherent descent, and separate reconstruction, definability, computability, and ontological reduction. Explicit counterexamples delimit these implications. The application to consciousness depends on independently justified semantic and modal premises.

1. Scope and interpretation

We ask whether a specified observational interface determines specified experiential distinctions. We treat representation of experience by a set, predicate, or higher type as a hypothesis requiring an adequacy argument.

We distinguish public records from complete physical descriptions, and actual from physically or metaphysically possible situations. Changing this domain changes a supervenience claim. Nonreconstruction in one model does not establish nonphysicality in every interpretation.

We use classical sets in Sections 2 and 7, an elementary topos in Section 3, and an ∞ -topos in Section 5. We require small indexing categories, enlarging universes when needed. We use ordinary equalities in ordinary categories and homotopy pullbacks and mapping spaces Map in ∞ -categories.

We take philosophical motivation from Chalmers [1, 2]. We recognize that van Benthem [3, Sections 1 and 3] already relates Beth definability to supervenience. We contribute an expository synthesis and separation of hypotheses, including higher coherence counterexamples. We claim no new density, descent, or definability theorem.

2. Reconstruction and observational refinement

Definition 2.1. We specify sets S, D, Q , a surjection $o : S \twoheadrightarrow D$, and a function $e : S \rightarrow Q$. We interpret S as admissible situations, D as attainable observational records, and Q as the distinctions under investigation. We call $r : D \rightarrow Q$ a reconstruction if $e = r \circ o$. We write

$$K_o = S \times_D S, \quad \ker(o) = \{(s, t) : o(s) = o(t)\}.$$

Theorem 2.2 (fiber criterion). We have

$$\exists! r : D \rightarrow Q \ (e = r \circ o) \iff \ker(o) \subseteq \ker(e).$$

Proof. We obtain the forward implication by applying r to equal observations. Conversely, we define $R \subseteq D \times Q$ by

$$R(d, q) \iff \exists s \in S \ (o(s) = d \wedge e(s) = q).$$

We obtain existence of a value for each d by surjectivity and uniqueness by fiberwise constancy. Thus R is the graph of a function r . We have $r(o(s)) = e(s)$. Any other reconstruction agrees at every d because o is onto. We require no choice of representatives. $\square$

We therefore obtain a concrete obstruction from $o(s) = o(t)$ and $e(s) \neq e(t)$. We obtain a reconstruction for a nonsurjective observation map only on its image; extension outside the image can fail or be nonunique.

Proposition 2.3 (refinement). We suppose $o = m \circ p$ for surjections $p : S \twoheadrightarrow P$ and $m : P \twoheadrightarrow D$. If e reconstructs through o , it reconstructs through p . Failure through p implies failure through o ; the converse need not hold.

Proof. We compose $r : D \rightarrow Q$ with m to reconstruct through p . We obtain the second implication by contraposition. For the failed converse, we take

$$S = \{0, 1\}^2, \quad p = \text{id}_S, \quad o(a, b) = a, \quad e(a, b) = b.$$

We reconstruct through p using e , while $(0, 0)$ and $(0, 1)$ obstruct reconstruction through o . We may interpret both coordinates as ordinary physical bits. $\square$

We cannot infer failure of physical determination from an incomplete measurement, or from individual finite procedures without analyzing their joint information.

3. Internal reconstruction in a topos

Theorem 3.1 (effective quotient). We let $\mathcal{E}$ be an elementary topos, let $o : S \twoheadrightarrow D$ be an epimorphism, and write $p_0, p_1 : K_o \rightrightarrows S$ for its kernel pair. For any $e : S \rightarrow Q$, we have a unique factorization through o exactly when

$$e \circ p_0 = e \circ p_1.$$

Proof. We use the standard exactness property that every epimorphism in a topos is the coequalizer of its kernel pair [4]. We obtain existence from this coequalizer property and uniqueness from epimorphic cancellation. We obtain necessity by composing $op_0 = op_1$ with a proposed reconstruction. $\square$

We may express the condition in the internal language as

$$\forall s, t : S, \quad o(s) = o(t) \Rightarrow e(s) = e(t).$$

We do not replace this internal statement by a test on global points alone: a topos need not have enough global points. Externally, we can test the equality using all generalized elements $T \rightarrow K_o$. We also distinguish a constructive witness of disagreement from the bare negation of a universal statement; intuitionistic logic does not generally identify those forms.

We can equivalently take the image $R \hookrightarrow D \times Q$ of $\langle o, e \rangle$. Its projection to D is epi and is also mono exactly when the relation is single-valued, hence the graph of a reconstruction. We require no section of o .

4. Categories of elements and adequate probes

We let $\mathcal{C}$ be small and $X, Y : \mathcal{C}^{op} \rightarrow \mathbf{Set}$. We define $\int X$ to have objects (c, x) with $x \in X(c)$ and arrows

$$f : (c, x) \longrightarrow (d, z), \quad f : c \rightarrow d, \quad X(f)(z) = x.$$

We write $U_X : \int X \rightarrow \mathcal{C}$ for the projection. We obtain a discrete fibration: for every arrow $f : c \rightarrow d$ and object (d, z) there is a unique lift with target (d, z) .

Proposition 4.1. We have natural bijections

$$\text{Nat}(X, Y) \cong \text{Fun}_{\mathcal{C}}(\int X, \int Y) \cong \lim_{(c,x) \in (\int X)^{op}} Y(c).$$

Proof. We send $\alpha : X \Rightarrow Y$ to $(c, x) \mapsto (c, \alpha_c(x))$, sending an arrow over f to the arrow over the same f . We obtain its existence from naturality. Conversely, a functor strictly over $\mathcal{C}$ defines the values $\alpha_c(x)$ on objects, and its action on arrows forces naturality. We obtain mutual inverses because lifts in the discrete fibrations are unique. We identify the limit with families $y_{c,x} \in Y(c)$ satisfying $Y(f)(y_{d,z}) = y_{c,x}$ for every arrow above. $\square$

We can equally regard these data as sections of the pullback of U_Y along U_X . We recover the density presentation

$$X \cong \text{colim}_{(c,x) \in \int X} y(c), \quad y(c) = \mathcal{C}(-, c).$$

Indeed, for every Y , we compute

$$\text{Hom}(\text{colim}_{\int X} y(c), Y) \cong \lim_{(\int X)^{op}} Y(c) \cong \text{Nat}(X, Y).$$

We identify the isomorphism by its component formula $[c, x, f : a \rightarrow c] \mapsto X(f)(x)$. We recover $z \in X(a)$ as $[a, z, \text{id}_a]$. We obtain this standard density argument from the Yoneda lemma; see [5, Proposition 1.1].

We now specify a class of probes by a functor $J : \mathcal{A} \rightarrow \mathcal{C}$, where $\mathcal{A}$ is small and $\mathcal{C}$ is locally small and cocomplete. We define

$$N_J(X)(a) = \mathcal{C}(Ja, X), \quad R_J(F) = \text{colim}_{(a,x) \in \int F} Ja.$$

Theorem 4.2 (density criterion). We have $R_J \dashv N_J$. We have full faithfulness of N_J exactly when every counit

$$\varepsilon_X : R_J N_J(X) \rightarrow X$$

is an isomorphism.

Proof. We compute maps from the displayed colimit into X as compatible maps $Ja \rightarrow X$, equivalently natural transformations $F \Rightarrow N_J X$. This gives the adjunction. If each counit is invertible, then

$$\mathcal{C}(X, Y) \cong \mathcal{C}(R_J N_J X, Y) \cong \text{Nat}(N_J X, N_J Y).$$

Conversely, under full faithfulness we have natural bijections

$$\mathcal{C}(R_J N_J X, Y) \cong \text{Nat}(N_J X, N_J Y) \cong \mathcal{C}(X, Y),$$

so Yoneda identifies the counit as an isomorphism. $\square$

We call such a probe functor dense. We then recover objects up to isomorphism and recover all morphisms. Failure of full faithfulness need not itself exhibit two nonisomorphic objects

with identical observations: it can concern morphisms instead. Full Yoneda is fully faithful, so an assertion of failure of every morphism-based description cannot follow inside the proposed category. We must specify restricted probes or challenge the adequacy of the entire mathematical representation.

5. Coherent reconstruction in an ∞ -topos

We let $\mathcal{E}$ be an ∞ -topos and $o : S \rightarrow D$ an effective epimorphism. We form its Čech nerve

$$C_n(o) = \underbrace{S \times_D \cdots \times_D S}_{n+1 \text{ factors}}.$$

We use effectiveness to identify $D \simeq |C_\bullet(o)|$ [6, Section 6.2.3].

Theorem 5.1 (coherent descent). For any $Q \in \mathcal{E}$, we have

$$\mathrm{Map}_{\mathcal{E}}(D, Q) \simeq \lim_{[n] \in \Delta} \mathrm{Map}_{\mathcal{E}}(C_n(o), Q).$$

Proof. We apply the universal property of the colimit $D \simeq |C_\bullet(o)|$: mapping out of a colimit gives the limit of mapping spaces. $\square$

Definition 5.2. For a specified $e : S \rightarrow Q$, we define the space of reconstructions by the homotopy fiber

$$\mathrm{Rec}_o(e) = \mathrm{Map}(D, Q) \times_{\mathrm{Map}(S, Q)} \{e\}.$$

We specify a point by (r, h) with $h : r \circ o \simeq e$. We can equivalently extend e to a complete coherent descent datum. For a fixed complete descent datum, the space of corresponding lifts is contractible, by Theorem 5.1. For the bare degree-zero datum e , the space $\mathrm{Rec}_o(e)$ can be empty or have several components or higher homotopy.

Corollary 5.3. If Q is 0-truncated, we recover the equalizer

$$\mathrm{Map}(D, Q) \cong \mathrm{Eq}(\mathrm{Map}(S, Q) \rightrightarrows \mathrm{Map}(S \times_D S, Q)).$$

Proof. We have discrete mapping spaces into Q . A compatible cone to the resulting cosimplicial sets is determined by its degree-zero value satisfying the two degree-one restrictions; higher compatibility follows from the simplicial identities. We therefore obtain an empty or singleton reconstruction space for each e . $\square$

Example 5.4 (nonunique higher reconstructions). We take discrete groups G, H , their one-object classifying groupoids BG, BH , and the effective epimorphism $o : * \rightarrow BG$ in spaces. We choose the basepoint map $e : * \rightarrow BH$. We have

$$\mathrm{Rec}_o(e) \simeq \mathrm{Map}_*(BG, BH) \simeq \mathrm{Hom}_{\mathrm{Grp}}(G, H)_{\mathrm{disc}}.$$

We prove the last equivalence using groupoids. A functor $BG \rightarrow BH$ specifies a homomorphism. A natural isomorphism conjugates it by an element of H ; including the chosen identification at the basepoint removes that conjugation freedom. We obtain a discrete groupoid of homomorphisms. For $G = H = C_2$ we obtain two points, represented by the trivial and identity homomorphisms. We thus have two inequivalent reconstructions of the same e , although o is an effective epimorphism. We cannot extend set-level uniqueness to arbitrary higher targets.

Example 5.5 (an obstruction at degree two). We take $G = C_2 = \{1, t\}$ and target BC_4 , writing C_4 additively. We give a degree-one candidate $a(1) = 0$, $a(t) = 1$. All arrows have the required endpoints. Coherence on two-simplices would require

$$a(gh) = a(g) + a(h).$$

At (t, t) we instead have $a(t^2) = 0$ and $a(t) + a(t) = 2$ in C_4 . We therefore cannot extend this particular degree-one candidate to coherent descent. We do not conclude that every choice is obstructed: $a(t) = 0$ and $a(t) = 2$ give genuine homomorphisms. We thereby distinguish failure of a chosen coherence from nonexistence of every reconstruction.

6. Sections, dependent data, and gluing

Proposition 6.1. In a category with products we have

$$\mathcal{E}(D, Q) \cong \Gamma_D(D \times Q \xrightarrow{\pi_D} D),$$

where we write Γ_D for sections over D .

Proof. We send r to $\langle \text{id}_D, r \rangle$ and a section s to $\pi_Q s$. We obtain inverse assignments by the product universal property. $\square$

We obtain the analogous equivalence of spaces in an ∞ -topos. For a dependent family $p : E \rightarrow D$, we instead ask for a section of p ; we do not identify this with choosing an arbitrary map to a fixed Q . When we regard a set D as a discrete category, the Grothendieck construction of a family (E_d) gives its disjoint union over D . The constant family gives $D \times Q \rightarrow D$.

We distinguish local inhabitation from a global section. For example, the double covering $z \mapsto z^2$ of the circle has local continuous sections and two points in each fiber, but no global continuous section. If s were a section, we would have

$$2 \deg(s) = \deg(\text{id}_{S^1}) = 1,$$

which is impossible in $\mathbb{Z}$. We may express this as a sheaf of local lifts with local sections but no global element. We use this example to separate gluing from local existence; we draw no conclusion about consciousness from the topology alone.

7. Definability and computation

We fix a classical first-order language L , an additional k -ary predicate P , and a theory T in $L \cup \{P\}$. We call P explicitly L -definable when some parameter-free L -formula $\varphi(\bar{x})$ satisfies

$$T \models \forall \bar{x} (P(\bar{x}) \leftrightarrow \varphi(\bar{x})).$$

Proposition 7.1 (two-expansion obstruction). If (M, P_0) and (M, P_1) satisfy T with the same L -reduct and $P_0 \neq P_1$, then P is not explicitly L -definable.

Proof. We would otherwise have $P_0 = \varphi^M = P_1$. $\square$

We say that T implicitly defines P when any two such expansions agree on P . By Beth's theorem [7], implicit and explicit definability coincide for classical first-order theories. We do not assert this for arbitrary logics or for a chosen restricted class of models. To connect a philosophical supervenience thesis with Beth's theorem, we must justify the language, the theory, the comparison of reducts, and the quantification over all its models [3]. We cannot infer indefinability from an unexplained sense of irreducibility.

We obtain Proposition 7.1 trivially by adding an unconstrained predicate. An experiential application requires independent justification of the expansions as possibilities, the predicate as tracking real differences, and the reduct as capturing the intended physical information.

Proposition 7.2 (determination does not imply computation). We can have a unique reconstruction that is arithmetically definable but not computable.

Proof. We take $S = D = \mathbb{N}$, $o = \text{id}$, $Q = \{0, 1\}$, and $e = \chi_K$, where K is the diagonal halting set for a fixed effective numbering of programs. We have the unique set-theoretic reconstruction $r = e$. We define $K(n)$ arithmetically by the existence of a finite halting computation record. If we could compute χ_K , we could define a program that halts on input n exactly when program n on input n does not halt, and evaluate it at its own index, a contradiction. We thus obtain definability and reconstruction without computability, following the standard diagonal method [8]. $\square$

We also distinguish set reconstruction from continuity. We take $S = \mathbb{R}$ with the discrete topology, $D = \mathbb{R}$ with its usual topology, $o = \text{id}$, and $e : S \rightarrow \{0, 1\}$ to be the indicator of $[0, \infty)$. We have a continuous surjection o , continuous e , and singleton fibers. We obtain a unique set reconstruction, but it is discontinuous. We do not contradict Theorem 3.1: this o is not a quotient map, and **Top** is not a topos. We must specify the category of admissible reconstructions.

8. The conditional philosophical inference

Definition 8.1. We stipulate a map $p : S \rightarrow P_{\text{phys}}$ representing complete physical information over a specified domain S of possibilities. We formulate phenomenal supervenience relative to this interpretation as

$$\forall s, t \in S, \quad p(s) = p(t) \Rightarrow e(s) = e(t).$$

Theorem 8.2 (conditional failure of supervenience). If we justify admissible s, t with $p(s) = p(t)$ and $e(s) \neq e(t)$, then no reconstruction $P_{\text{phys}} \rightarrow Q$ exists and the stipulated supervenience thesis fails.

Proof. We apply Theorem 2.2 to p . $\square$

We have already supplied a pair identical physically but different phenomenally. The mathematics does not establish its modal possibility. To infer idealism or reject every physicalist ontology, we also need premises connecting those ontologies to this supervenience condition. Naming p physical supplies no such connection.

Proposed inference	What we require or can conclude
Equal reports imply equal experiences	We require the report map to satisfy the fiber criterion.
Unsuccessful neural decoding implies nonphysicality	We require a completeness premise; the two-bit model blocks the bare inference.
Local experiential choices give a global choice	We require gluing and coherence; local inhabitation alone does not suffice.
A symbolically specified Q proves all experience is symbolic	We require an adequacy argument for that representation.
Every symbolic account is impossible	We require a defined class of languages and an obstruction for each member.

We distinguish realism about experience, idealism, and denial of phenomenal reality. We establish relative reconstruction results and identify the premises needed for stronger ontological conclusions.

9. Evidence, limitations, and authorship

We provide proofs and finite verification of 5,518 bounded cases and 16 cyclic-group comparisons. We claim no empirical results or proof-assistant certification. Bounded verification does not replace proofs or validate experiential interpretations.

We use AI assistance for drafting, exposition, source checking, and artifact preparation. We use the byline Anonymous at the responsible submitter’s direction; responsibility for claims and interpretation remains with the submitter. We distinguish submission from acceptance or peer review.

We leave three substantive tasks open: constructing an independently defensible experiential model, identifying an observational interface with a justified completeness claim, and deriving a nontrivial obstruction in that model. We offer this note as a framework for those tasks.

References

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