

Corrections to Operadic Autopoiesis: Conservativity, Inhabitation, Budgets, and Viability

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Abstract

We correct mathematical statements in *Organisms, Viruses, and Computers: Closure, Operadic Autopoiesis, and Thermodynamic Viability*. Conservativity does not recover objects from equivalent images; noninitiality does not imply internal inhabitation; and a fixed cost sublevel need not be an operad. We prove that taking the generated image of a coupling pushout into a fixed ambient operad loses the interface parameter. We identify additional hypotheses required to interpret seeded production closure as regeneration and predecessor fixed points as sustained trajectories. A finite, resource-aware transition semantics gives a corrected policy-realization theorem and an executable mutual-repair example. The standard invariant-obstruction arguments survive. These corrections concern mathematical content and do not infer the source manuscript’s authorship or preparation history.

1. Scope and conventions

The target is the three-essay manuscript, edition 1.0, attached to the published record [1], not merely its web synopsis. We denote its essays by I, II, and III; all target section numbers refer to that edition. Its publication label is *Editorially reviewed*. Our results do not depend on that label. The source archive SHA-256 is recorded in the accompanying correction map.

Ordinary categories are regarded as infinity-categories through their nerves. Statements about an arbitrary infinity-topos $\mathcal{E}$ use its internal logic; explicitly finite examples use ordinary sets. We fix universes so the subobject lattices used below are sets in the working universe. A physical interpretation is additional data, not a consequence of choosing $\mathcal{E}$.

The paper distinguishes false mathematical statements, constructions requiring extra hypotheses, and conditional modeling claims. It supplies no biological experiment, exhaustive definition of life, or impossibility theorem for artificial organisms. Numbered results below are proved here; foundational constructions are attributed to [2-6].

2. Invariant arguments that remain valid

Proposition 2.1 (images and closure). Suppose structured equivalences preserve diagrams $D \xrightarrow{d} R \xleftarrow{p} P$. Then the predicate $\text{im}(d) \leq \text{im}(p)$ is invariant. A geometric inverse-image functor preserves this inequality. Pullback along an effective epimorphism reflects it.

Proof. Equivalences transport the image factorization and induce order isomorphisms on subobjects. Geometric inverse images preserve finite limits and colimits, hence monomorphisms and the effective epimorphisms computed from Cech nerves. They therefore preserve images and their inclusions. Effective-epimorphic pullback reflects subobject order: the pullback of the intersection of two subobjects equals the first precisely when the first is contained in the second, and the corresponding equivalence descends. Apply this in the relevant slice over the context. These are the standard image and descent properties of an infinity-topos [3, §6.2.3]. $\square$

Proposition 2.2 (fixed-point transport). Let $F : L \rightarrow M$ be an order isomorphism of complete lattices and let $F\Phi = \Phi'F$. For monotone Φ, Φ' , one has $F(\mu\Phi) = \mu\Phi'$ and $F(\nu\Phi) = \nu\Phi'$.

Proof. The conjugacy bijects fixed points and preserves their order, so it transports the least and greatest ones. Their existence is the lattice fixed-point theorem [4]. $\square$

Consequently, different values of a genuinely preserved invariant obstruct structured equivalence. This valid principle does not establish that a proposed biological system has a specified invariant value.

3. Conservativity and recovery of objects

Definition 3.1. A functor $F : \mathcal{C} \rightarrow \mathcal{D}$ is conservative if $F(f)$ being an equivalence implies that f is an equivalence, for every morphism f . Say separately that F reflects object-equivalence classes if $F(X) \simeq F(Y)$ implies $X \simeq Y$.

Theorem 3.2 (independence). Neither property in Definition 3.1 implies the other.

Proof. Let $\mathcal{C}$ be the discrete category on two objects and $F : \mathcal{C} \rightarrow 1$ the unique functor. Every morphism of $\mathcal{C}$ is an identity, so F is conservative. Its two nonisomorphic objects have the same image. Conversely, let $\mathcal{C}$ have one object and endomorphism monoid $\{1, e\}$ with $e^2 = e \neq 1$. The unique functor to 1 reflects object-equivalence classes because there is only one source object. It is not conservative: e is noninvertible and its image is the identity. $\square$

A familiar example is $U : \mathbf{Grp} \rightarrow \mathbf{Set}$. A bijective homomorphism has a homomorphic inverse, so U is conservative [2, Lemma 5.6.1]. Yet the underlying sets of C_4 and $C_2 \times C_2$ are bijective while their element orders distinguish the groups.

Proposition 3.3 (sufficient lifting condition). If F is conservative and every equivalence $F(X) \simeq F(Y)$ is homotopic to $F(f)$ for some $f : X \rightarrow Y$, then F reflects object-equivalence classes. In particular, a fully faithful functor has both properties.

Proof. Lift the given equivalence and apply conservativity. For a fully faithful functor, equivalences and their inverses lift through the equivalences of mapping spaces; the inverse identities lift as well. $\square$

Thus II, Theorem 47.1, is false. The arguments in I §18 and II §35 establish at most a failure to recover objects, unless a noninvertible morphism with invertible image is exhibited. Forgetting structure alone is not evidence of nonconservativity.

There is an important qualification: II, Proposition 37.2, concerns an actual reflective localization $L : \mathcal{C} \rightleftarrows \mathcal{D} : i$, with fully faithful i . If every unit $X \rightarrow iLX$ is an equivalence, L is an equivalence and is conservative. Otherwise choose a nonequivalence unit. Then $X \not\simeq iLX$, since any object equivalent to a local object is local, but $LX \simeq LiLX$. Hence that proposition is recoverable using the localization structure; it does not follow from the false general criterion.

4. Noninitiality, inhabitation, and witnesses

For $A \in \mathcal{E}$, distinguish

$$A \neq 0, \quad \text{im}(A \rightarrow 1) = 1, \quad s : 1 \rightarrow A.$$

The middle condition expresses the truth of internal propositional inhabitation. The last supplies a global witness. A section implies effective-epimorphic inhabitation, which implies noninitiality in a nondegenerate topos; neither converse holds generally.

Proposition 4.1 (nonzero kernel without inhabitation). There is an infinity-topos and an identity-transition system whose greatest viable fixed point is noninitial but neither internally inhabited over the terminal context nor globally pointed.

Proof. In $\text{Shv}(\mathbb{R}; \mathcal{S})$, let $A \hookrightarrow 1$ be the subterminal sheaf represented by the open set $(0, 1)$. It is noninitial. Its image in 1 is the same proper subobject, and it has no section over all of $\mathbb{R}$. Take state object and viability object both equal to A , with one identity control. On $\text{Sub}(A)$ the predecessor and viability operators are identities. Their greatest fixed point is A . $\square$

Even internal inhabitation need not provide a global section: in $\text{Fun}(BG, \mathcal{S})$ for $G = C_2$, the regular G -set $G \rightarrow 1$ is an effective epimorphism, but a section would be a fixed point of a free action.

Accordingly III, Definition 20.1, cannot replace inhabitation by $\nu\Psi \neq 0$. For a specified context W , a physical claim can require $s_0 : W \rightarrow \nu\Psi$ together with policy data over W . If only local existence is intended, require the appropriate image in W to equal W and state that weaker interpretation.

5. Seeded generation is not regeneration

Proposition 5.1 (seed retention). On any complete suboperad lattice with a chosen E , the operator $\Phi(A) = E \vee A$ is monotone and inflationary, preserves filtered joins, and has $\mu\Phi = E$.

Proof. Joining with E preserves order and nonempty joins, and $A \leq E \vee A$. A fixed point is exactly a suboperad containing E ; the least is E . $\square$

This satisfies the order-theoretic hypotheses of II §§11-13. If $R \leq E$, its closure test succeeds without introducing a single new production capability. The point is not that a least fixed point is illegitimate, but that initial availability does not certify replacement of its physical realizers. An adequacy axiom must identify the regeneration obligations for every required piece of machinery, its degradation, and the processes that discharge those obligations. A sufficiently rich R may do so; the displayed inclusion alone does not.

Closure also depends on requirements: for suboperads $A < B$, take $G_X = G_Y = A$, $R_X = A$, and $R_Y = B$. One object passes and the other fails, although their generated operads are identical. The inference to unequal generated operads in II §31 therefore needs an additional hypothesis.

Finally, separate witnesses do not supply a common schedule. Let two maintenance obligations each require the sole unit of energy before the same deadline. Each is separately feasible and their simultaneous conjunction is not. Formally, $\forall r \exists \pi_r$ does not imply $\exists \pi \forall r$. A viable trajectory must discharge the same production obligations counted by the closure doctrine.

6. Resource grading and realizable costs

Here costs are ordinary extended nonnegative real numbers with their usual order. If a Lawvere reversed order is used instead, all comparisons must be translated explicitly [5]. Assume a realization relation $p \models f$ and compatible substitution of actual realizers with subadditive cost Θ .

Definition 6.1. Set

$$\mathcal{O}_{\leq b}^{\text{act}} = \{f \mid \exists p \models f, \Theta(p) \leq b\}.$$

Proposition 6.2 (graded composition). Compatible realizers induce

$$\mathcal{O}_{\leq b_0}^{\text{act}} \times \prod_i \mathcal{O}_{\leq b_i}^{\text{act}} \longrightarrow \mathcal{O}_{\leq b_0 + \sum_i b_i}^{\text{act}}$$

on each matching profile. A fixed sublevel need not be a suboperad.

Proof. Substitute the witnessing realizers and use subadditivity. For the second assertion, take the free unary operad on f , with unique word realizations and cost $\Theta(f^n) = n$. Budget 1 admits the identity and f , but not $f \circ f$. $\square$

Proposition 6.3 (infimum versus attainment). For $c(f) = \inf_{p \models f} \Theta(p)$, the condition $c(f) \leq b$ need not imply actual b -feasibility. If $c(f) < b < \infty$, actual b -feasibility follows.

Proof. Realizers of costs $1 + 1/n$, $n \geq 1$, have infimum 1 but none meets budget 1. For strict slack, if no realizer cost were below b , then b would be a lower bound, contradicting $c(f) < b$. $\square$

Under the compatibility assumption, choosing finitely many realizers within arbitrary positive slack proves the numerical inequality

$$c(f \circ (g_1, \dots, g_n)) \leq c(f) + \sum_i c(g_i).$$

The inequality survives nonattainment. Exact-budget realization does not. Separate optimization also requires compatible physical interfaces: abstractly matching operation labels do not establish that independently chosen physical realizers compose.

III §§9-11 should therefore use graded operations or resource-state transitions. A supply rate can support repeated operations, but only with specified durations, storage and replenishment. It is not interchangeable with a finite total-energy budget.

7. What the generated image of a pushout forgets

Theorem 7.1 (collapse to the join). Let E, H be suboperads of a fixed operad O , and let $E \leftarrow I \rightarrow H$ commute over O . Suppose the pushout exists and $J = E \vee H$ is a suboperad through which the common-interface agreement lifts. Then

$$\langle \text{im}(E \amalg_I H \rightarrow O) \rangle = J.$$

The hypotheses hold for ordinary colored operads with monic inclusions. They also hold in higher models where the suboperad inclusions are monomorphisms and the indicated pushout and generated subobjects exist.

Proof. The generated image contains the images of E and H , hence contains J . Both maps to O factor through J , and their agreement lifts by hypothesis (automatically along a monomorphism). The pushout universal property gives $E \amalg_I H \rightarrow J$. Thus its generated image in O is contained in J . $\square$

For fixed embeddings into O , the result is independent of I whenever the hypotheses hold. This corrects the interpretation of II §19, not the existence of mixed operations in the raw pushout. One may retain the full pushout and its realization map, or make interface permission part of the colors and composition laws. Structured cospans provide one established interface-sensitive setting [6].

For a concrete unary example, let O be the category $a \xrightarrow{f} b \xrightarrow{g} c$. Its subcategories generated separately by f and g have a coproduct over the empty category with two distinct copies of b . That coproduct has no mixed path gf . Mapping into O identifies the two copies; generating a subcategory adds gf . Interface information has been discarded by the latter step.

8. Viability, choice, and uncertainty

For a controlled relation T , set $\Psi(A) = V \cap \text{Pre}(A)$, where predecessor means existence of an admissible control into A . Monotonicity yields $K = \nu\Psi$ [4]. The equation $K = \Psi(K)$ establishes a serial successor relation on K .

Proposition 8.1 (seriality and paths). In classical sets with dependent choice, every $s_0 \in K$ starts an infinite admissible path in K . Internally, the same conclusion follows from a specified control selector on K and recursion on the natural numbers.

Proof. Apply dependent choice to the relation consisting of a control and its successor, starting at s_0 . With a selector, define the next state by recursion instead. $\square$

A bare serial relation does not supply that selector in an arbitrary topos. The assertion that every serial relation admits such a sequence is precisely a dependent-choice principle. In continuous time, an infinite action sequence additionally needs unbounded elapsed time and existence of trajectories; infinitely many events in finite time are insufficient.

The stochastic semantics must be chosen separately. For a kernel P , almost-sure one-step safety uses $\exists u P(s, u, A) = 1$, not merely a successor of positive probability. In the two-state chain with survival probability $q \in (0, 1)$ at every step and an absorbing failure state, a possible surviving successor exists at every step, but infinite survival has probability $\lim_n q^n = 0$. Robust control instead uses a predecessor of the form $\exists u \forall w, T(s, u, w) \in A$. Each operator is monotone; their meanings and kernels differ. Thus the stochastic extension in III §15 requires a new specification, even though a lattice argument remains available.

9. A corrected finite realization theorem

We give a sufficient, deliberately restricted semantics rather than a universal definition of life. Let S be a finite set of configurations, U a finite ordered control set, and $T : S \times U \rightarrow S$ a specified deterministic transition function. Each transition advances time by one unit. Configurations include resource stocks, machine status, and deadline counters for a finite list of regeneration obligations. The transition table must encode resource spending and permitted environmental replenishment. Its interpretation as physical repair is a declared modeling hypothesis.

Let $V \subseteq S$ impose resource bounds, machine-function conditions and finite deadlines. Assume each obligation counter resets only when a transition records a valid realization of that regeneration obligation, and otherwise increments each tick. Define

$$K_0 = V, \quad K_{n+1} = V \cap \{s : \exists u, T(s, u) \in K_n\}.$$

Theorem 9.1 (finite policy realization). The descending sequence stabilizes at $K = \nu\Psi$ after at most $|S|$ strict decreases. For a specified $s_0 \in K$, choose the first control $u(s)$ with $T(s, u(s)) \in K$. This policy generates a single infinite trajectory satisfying the encoded resource constraints and every regeneration deadline. Conversely, every infinite trajectory inside V starts in K .

Proof. Monotonicity and $K_1 \subseteq K_0$ make the sequence descending. A strict decrease removes at least one state. Its stable value is fixed; induction shows every fixed point is contained in every K_n , so this is the greatest fixed point. At each state of K a suitable control exists; finite ordered search selects it without an additional choice axiom. Recursion yields a trajectory in $K \subseteq V$. The counter-update rule and bounded counters ensure each obligation is discharged within its deadline. For the

converse, the set of states appearing on a viable infinite trajectory is postfixed, hence contained in the greatest fixed point. $\square$

Corollary 9.2 (transport). A bijection of states and controls preserving transitions, viability, resource accounting, obligation labels and time units transports K and feasible policy trajectories.

Proof. The induced powerset order isomorphism conjugates the predecessor operators; apply Proposition 2.2 and transport the trajectory pointwise. $\square$

This is stronger than separately asserting that each repair is generable and that some unrelated trajectory is viable. It supplies a common execution witness. The theorem is finite and deterministic; it makes no claim to solve arbitrary stochastic or internal higher-categorical synthesis.

Example 9.3 (mutual repair). A state is (b, a_1, a_2) with $b \in \{0, 1\}$ and ages $a_i \in \{0, 1, 2\}$. Every tick, supply $h \in \{0, 1\}$ changes the available resource to $\min(1, b + h)$. Controls are idle or repair one module at cost one; its functional partner is the declared repair realizer. Both ages advance, and the repaired age is reset to zero. A successor with an age above two is inadmissible. The script enumerates all 18 states. For $h = 1$, the kernel has 16 states (all except those with both ages two). For $h = 0$, the kernel is empty. Repeatedly repairing the older module gives an infinite witness in the supplied case. This is a mathematical transition model, not a measured cell or fabrication device.

10. Energetic obstruction with explicit assumptions

Fix a class Π_V of otherwise viable policies and a nonnegative cumulative exergy requirement $C_\pi(T)$. Set $\inf \emptyset = +\infty$ and

$$m = \inf_{\pi \in \Pi_V} \limsup_{T \rightarrow \infty} C_\pi(T)/T.$$

Budget-feasible policies must satisfy $C_\pi(T) \leq B_0 + E_{\text{in}}(T)$ for every T , with $B_0 < \infty$. These are additional accounting assumptions.

Theorem 10.1 (power obstruction). If $\limsup_{T \rightarrow \infty} E_{\text{in}}(T)/T \leq P < m$, no indefinitely viable policy is budget-feasible.

Proof. For any such policy, division by T and the finite initial store imply $\limsup C_\pi(T)/T \leq P$. The defining infimum implies $m \leq \limsup C_\pi(T)/T$, a contradiction. If Π_V is empty there is already no viable policy. $\square$

This supplies the missing accounting for III, Theorem 24.1. Strict positivity of m is not a consequence of nonnegative entropy production. Results such as [7] prove maintenance bounds for specified Markov dynamics and target distributions; [8] supplies explicit chemical-network thermodynamic balances. An invariant involving average power must preserve the physical clock, policy class and valuation, not only a reachability relation. The precise hypotheses of either reference cannot be replaced by an unspecified cost decoration.

11. Consequences for the biological interpretation

The corrected obstruction statement is conditional: if an equivalence preserves a specified organizational predicate and two models have different truth values, the models are not equivalent in that category. Demonstrating those different values for a bacterium, virion and computer requires a common typing discipline, explicit boundaries, consistent treatment of feedstock versus machinery, and a justified requirement signature.

The abstract choice of an infinity-topos is legitimate. A concrete physical instance still needs an interpretation of its objects, morphisms, homotopies and observables. Several obstruction lemmas already hold for discrete data; their higher-categorical formulation alone supplies no additional biological evidence. Heredity and lineage must be defined before they can serve as further invariant coordinates. A full category of systems also needs morphisms and comparison functors, not merely a list of object fields and proposed equivalences.

The source correctly allows artificial self-maintaining systems and does not claim a complete classifier of life. Those qualifications survive. The present corrections neither identify organisms with ordinary computers nor prove an absolute separation of all possible computational and biological realizations.

12. Artifacts, limitations, and preparation

The accompanying archive contains this complete Markdown manuscript, LaTeX source, a PDF, an offline MathML reading edition, a numbered correction map, bibliographic metadata, a Python standard-library counterexample checker, its output, build instructions and checksums. The checker exhaustively verifies specified finite examples, including the viability kernels in Example 9.3. It is not a proof-assistant formalization of the general theorems or an empirical validation.

The correction map also records local edits: III §12 reverses an intended inclusion comparison; II §54 treats π_0 as a group without additional structure; and the effective-epimorphic descent arguments do not establish unrestricted claims about every localization. No novelty is claimed for the classical foundational results used here. The contribution is their precise application to the target statements and the explicit repair conditions.

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