

Organisms, Viruses, and Computers: Closure, Operadic Autopoiesis, and Thermodynamic Viability

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An (∞) -Topos-Theoretic Separation of Organisms, Viruses, and Computers

Abstract

We formulate a categorical distinction between living organisms, viruses, and computational machines inside an (∞) -topos E . The distinction is not identified with computational behavior, nor with material substrate alone. A physical system carries typed components, internally generated processes, required organizational resources, and interfaces to an environment.

For each organizational mode a , a system X has a demand morphism and an endogenous production morphism

$$d_a^X : D_a(X) \longrightarrow R_a(X), \quad p_a^X : P_a(X) \longrightarrow R_a(X).$$

The system is closed in mode a when $\text{im}(d_a^X) \leq \text{im}(p_a^X)$ in the subobject lattice of $R_a(X)$. Everything required as an organizational enabling condition is then generated by processes internal to the system. Matter and energy may still enter through explicitly external channels.

This yields three notions: autonomous closure, host-relative closure, and externally maintained machinery. A cellular organism is modeled as organizationally closed over an appropriate signature. A virus is not autonomously closed, but becomes closed after supplementation by host processes. An ordinary computer is not closed for fabrication, repair, and physical reproduction, though it may be closed for a restricted signature of computational operations.

These closure properties are homotopy invariant, preserved by equivalences and by geometric inverse-image functors, and satisfy descent. They are therefore genuine obstructions to equivalence. A material-support invariant is defined separately and shown to be logically independent of organizational closure. The analogy between a virus and a program arises only after a non-conservative forgetful functor erases structure needed to compare either of them with a cell.

1. Ambient (∞) -topos

Let E be an (∞) -topos. One may think of E as a category of homotopy-coherent sheaves of physical states, processes, or observations, over a site of spacetime regions, experimental contexts, or interaction contexts.

An (∞) -topos, rather than an ordinary category, supplies the following.

- arbitrary higher homotopies between realizations
- descent and locality
- an internal intuitionistic higher-order logic
- effective epimorphism–monomorphism factorizations
- a well-behaved theory of subobjects and images

The properties used below are the standard ones of (∞) -topoi in the sense of **Higher Topos Theory**.

Definition 1.1 Subobject

For $Y \in E$, a subobject of Y is a (-1) -truncated morphism $A \hookrightarrow Y$. Equivalence classes of such morphisms form a partially ordered set $\text{Sub}_E(Y)$. The order is $A \leq B$ if and only if $A \hookrightarrow Y$ factors through $B \hookrightarrow Y$. Internally, $A \leq B$ means that every element satisfying A also satisfies B .

Definition 1.2 Image

Every morphism $f : X \rightarrow Y$ admits a canonical factorization

$$X \twoheadrightarrow \text{im}(f) \hookrightarrow Y,$$

in which the first factor is (-1) -connected — equivalently, an effective epimorphism — and the second is (-1) -truncated. This is the (-1) -connected/ (-1) -truncated factorization system of an (∞) -topos. Hence $\text{im}(f) \in \text{Sub}(Y)$ is well defined up to a contractible space of choices.

2. Typed physical systems

We first encode material composition. Let $K \in E$ be an object of physical component types. In a sufficiently concrete model, points of K might include protein, lipid, DNA, silicon, copper, transistor, ribosome, and so on. No discreteness assumption is required.

Definition 2.1 Typed system

A K -typed physical system X consists in part of an object $C_X \in E$ of components together with a typing morphism $\tau_X : C_X \rightarrow K$. Its material support is the image

$$\text{Supp}(X) := \text{im}(\tau_X) \hookrightarrow K.$$

Thus $\text{Supp}(X)$ is the subobject of component-types actually instantiated by X .

Definition 2.2 Type-preserving equivalence

An equivalence $X \simeq Y$ of K -typed systems is required to contain an equivalence $f : C_X \xrightarrow{\sim} C_Y$ and a homotopy-commutative square

$$\begin{array}{ccc} C_X & \xrightarrow{f} & C_Y \\ \tau_X \downarrow & & \downarrow \tau_Y \\ K & = & K, \end{array}$$

together with the analogous coherent equivalences on the remaining structure. Commutativity says $\tau_X \simeq \tau_Y \circ f$.

3. The substrate invariant

Theorem 3.1 Material-support invariance

If $X \simeq Y$ as K -typed physical systems, then $\text{Supp}(X) \simeq \text{Supp}(Y)$ in $\text{Sub}(K)$.

Proof.

The equivalence supplies $f : C_X \xrightarrow{\sim} C_Y$ with $\tau_X \simeq \tau_Y \circ f$. Homotopic morphisms determine the same image subobject: image depends on the morphism only through its class in the homotopy category, via the canonical (-1) -connected/ (-1) -truncated factorization. Precomposition with an equivalence does not change that image, so

$$\text{im}(\tau_X) \simeq \text{im}(\tau_Y \circ f) \simeq \text{im}(\tau_Y).$$

Hence $\text{Supp}(X) \simeq \text{Supp}(Y)$. The general homotopy statement is Lemma 6.1.

Corollary 3.2 Substrate obstruction

If $\text{Supp}(X) \not\simeq \text{Supp}(Y)$, there is no equivalence $X \simeq Y$ in any category whose equivalences preserve the physical typing $\tau \rightarrow K$.

Differences of physical composition are therefore a genuine obstruction, provided composition is among the structure being preserved. A category that deliberately forgets τ cannot, by construction, detect the distinction.

4. Organizational signatures

Material composition is not the only relevant invariant. Let Σ be a collection of organizational modes. Typical elements might be

$$\Sigma = \{\text{boundary, catalysis, repair, translation, replication, regulation, } \dots \}.$$

The claim is not that every organism instantiates exactly this list. The signature Σ records which structural capacities a given theory treats as organizationally essential.

Generic matter and free energy need not belong to Σ . Living systems are thermodynamically open. The closure defined here is not closure to matter and energy. It is closer to Rosen's closure to efficient causation, and to the organizational closure of Maturana and Varela. Maturana characterized autopoietic organization as a network of component-producing processes that recursively regenerates the network and its boundary. Rosen's relational biology likewise separates material flows from internally regenerated organizational causes.

5. Organizational resource data

Definition 5.1 Σ -system

A Σ -system X consists, for each $a \in \Sigma$, of an organizational-resource object $R_a(X)$, a demand morphism

$$d_a^X : D_a(X) \longrightarrow R_a(X),$$

and an endogenous-production morphism

$$p_a^X : P_a(X) \longrightarrow R_a(X).$$

Here $D_a(X)$ enumerates organizational enablers required by X , and $P_a(X)$ enumerates organizational enablers produced by processes belonging to X . Write

$$\text{Req}_a(X) := \text{im}(d_a^X), \quad \text{End}_a(X) := \text{im}(p_a^X).$$

Both are subobjects of $R_a(X)$.

Definition 5.2 Closure in a mode

The system X is closed in mode a when $\text{Req}_a(X) \leq \text{End}_a(X)$. Equivalently, the demand image factors through the production image:

$$\text{Req}_a(X) \longrightarrow \text{End}_a(X) \hookrightarrow R_a(X).$$

Write $\text{Cl}_a(X)$ for this proposition.

Definition 5.3 Σ -closure

The system is Σ -closed when it is closed in every mode:

$$\text{Cl}_\Sigma(X) \iff \bigwedge_{a \in \Sigma} \text{Cl}_a(X) \iff \bigwedge_{a \in \Sigma} (\text{Req}_a(X) \leq \text{End}_a(X)).$$

This is an internal proposition of the theory.

6. Homotopy invariance

Lemma 6.1 Homotopic maps have the same image

If $d \simeq d' : D \rightarrow R$, then $\text{im}(d) \simeq \text{im}(d')$. The same holds for production maps.

Proof.

Image is determined by the canonical (-1) -connected/ (-1) -truncated factorization of the morphism represented in the homotopy category. Homotopic morphisms define equivalent objects of the arrow (∞) -category $\text{Fun}(\Delta^1, E)$, and therefore equivalent image factorizations. Their image subobjects coincide.

Corollary 6.2 Closure is not presentation-dependent

The truth value of $\text{Cl}_a(X)$ is invariant under all coherent higher homotopies in the presentation of X . The closure invariant is (∞) -categorical.

7. Closure is preserved by equivalence

Theorem 7.1 Organizational closure is invariant

Let $X \simeq Y$ be an equivalence of Σ -systems preserving the organizational-resource diagrams. Then $\text{Cl}_a(X)$ if and only if $\text{Cl}_a(Y)$, for every $a \in \Sigma$. Consequently $\text{Cl}_\Sigma(X)$ if and only if $\text{Cl}_\Sigma(Y)$.

Proof.

A structure-preserving equivalence supplies $R_a(X) \simeq R_a(Y)$, $D_a(X) \simeq D_a(Y)$, and $P_a(X) \simeq P_a(Y)$, under which the demand and production diagrams agree up to coherent homotopy. By Lemma 6.1 the image subobjects correspond:

$$\text{Req}_a(X) \simeq \text{Req}_a(Y), \quad \text{End}_a(X) \simeq \text{End}_a(Y).$$

An equivalence induces an isomorphism of the corresponding subobject posets and preserves their order. Therefore $\text{Req}_a(X) \leq \text{End}_a(X)$ if and only if $\text{Req}_a(Y) \leq \text{End}_a(Y)$.

Corollary 7.2 Organizational obstruction

If $\text{Cl}_a(X)$ and $\neg \text{Cl}_a(Y)$ for some mode a , then $X \not\simeq Y$ as Σ -systems.

This is the basic separation theorem. Equivalence cannot cross a difference of closure.

8. External supplementation

A virus is not handled by merely changing the context in which it is observed. The host contributes machinery. Base change and supplementation are different operations.

Suppose an external system H supplies additional a -resources along $q_a^H : Q_a(H) \rightarrow R_a(X)$. Form the combined supply $P_a(X) \sqcup Q_a(H) \xrightarrow{[p_a^X, q_a^H]} R_a(X)$.

Definition 8.1 Relative closure

The system X is closed in mode a relative to H when

$$\text{Req}_a(X) \leq \text{im}([p_a^X, q_a^H]).$$

Write $\text{Cl}_a(X \mid H)$.

Definition 8.2 Strict external dependence

The system X is strictly dependent on H in mode a when $\neg \text{Cl}_a(X)$ but $\text{Cl}_a(X \mid H)$. Thus X cannot generate all of its required organizational resources, while the coupled system can.

9. Image of combined production

Lemma 9.1 Image of a copairing

Let $p : P \rightarrow R$ and $q : Q \rightarrow R$. Then

$$\text{im}([p, q]) = \text{im}(p) \vee \text{im}(q)$$

in $\text{Sub}(R)$.

Proof.

Both p and q factor through $\text{im}([p, q])$, so $\text{im}(p) \leq \text{im}([p, q])$ and $\text{im}(q) \leq \text{im}([p, q])$. Hence their join is contained in the image of the copairing.

Conversely, $[p, q]$ factors through every subobject through which both p and q factor, and in particular through $\text{im}(p) \vee \text{im}(q)$. By minimality of the image, $\text{im}([p, q]) \leq \text{im}(p) \vee \text{im}(q)$.

Relative closure may therefore be rewritten

$$\text{Req}_a(X) \leq \text{End}_a(X) \vee \text{Host}_a(H),$$

where $\text{Host}_a(H) := \text{im}(q_a^H)$.

10. Base change and descent

Let $f^* : E \rightarrow F$ be the inverse-image functor of a geometric morphism of (∞) -topoi. It preserves finite limits and all small colimits.

Lemma 10.1 Inverse image preserves factorization

The functor f^* preserves effective epimorphisms, monomorphisms, and image factorizations.

Proof.

Effective epimorphisms are the morphisms whose Čech nerves are colimit diagrams. Since f^* preserves colimits and finite limits, it sends Čech nerves to Čech nerves, and effective epimorphisms to effective epimorphisms. Left exactness preserves (-1) -truncated morphisms, hence monomorphisms. Applying f^* to an effective-epimorphism/monomorphism factorization therefore yields the image factorization after base change.

Theorem 10.2 Geometric invariance

If $\text{Cl}_a(X)$ holds in E , then $\text{Cl}_a(f^* X)$ holds in F .

Proof.

From $\text{Req}_a(X) \leq \text{End}_a(X)$, apply f^* . Lemma 10.1 gives

$$f^* \text{Req}_a(X) \simeq \text{Req}_a(f^* X)$$

and the analogous identification for endogenous production. Since f^* preserves factorization through monomorphisms,

$$\text{Req}_a(f^* X) \leq \text{End}_a(f^* X).$$

11. Closure satisfies descent

Let $u : U \twoheadrightarrow B$ be an effective epimorphism. Pullback induces $u^* : \text{Sub}(B) \rightarrow \text{Sub}(U)$. Along an effective epimorphism this map is conservative on subobjects: it reflects order.

Theorem 11.1 Local-to-global closure

Suppose a system over B becomes organizationally closed after pullback along an effective epimorphism $u : U \twoheadrightarrow B$. Then it was already organizationally closed over B .

Proof.

The hypothesis is $u^* \text{Req}_a(X) \leq u^* \text{End}_a(X)$. Pullback along an effective epimorphism reflects the order of subobjects, so $\text{Req}_a(X) \leq \text{End}_a(X)$. Thus $\text{Cl}_a(X)$. Applying this to every $a \in \Sigma$ yields $\text{Cl}_\Sigma(X)$.

Corollary 11.2 A host is not a local context

Suppose $\neg \text{Cl}_a(V)$. There is no effective-epimorphic observational cover $U \twoheadrightarrow 1$ such that mere restriction to U makes V closed. If $\text{Cl}_a(u^* V)$ held, Theorem 11.1 would give $\text{Cl}_a(V)$.

Host dependence cannot be represented by localization or by a change of observational context. The host must contribute additional organizational morphisms $q_a^H : Q_a(H) \rightarrow R_a(V)$. Restriction and coupling are different.

12. Axiomatized cells

Let Σ_{org} contain modes such as boundary, catalysis, repair, biosynthesis, and replication.

Definition 12.1 Autopoietic Σ -object

An autopoietic Σ -object is a system B with $\text{Cl}_{\Sigma_{\text{org}}}(B)$. External material and energy maps $M_{\text{ext}} \rightarrow B$ are explicitly permitted. The condition says only that the organizational machinery named by Σ_{org} is internally regenerated.

This is not thermodynamic closure. It formalizes the core of autopoiesis and of closure to efficient causation. A single-celled organism is then modeled as an object

$$B \in \text{Aut}_{\Sigma}(E) \subseteq \text{Sys}_{\Sigma}(E).$$

That is an empirical modeling hypothesis, not a theorem of pure mathematics.

13. Axiomatized viruses

Viral replication depends on host-cell machinery: on host metabolism, translation, membranes, and biosynthetic resources.

Let V be a viral system and H a host.

Definition 13.1 Host-dependent replicator

The virus V is a host-dependent replicator if for some $a \in \Sigma_{\text{rep}}$,

$$\neg \text{Cl}_a(V) \quad \text{and} \quad \text{Cl}_a(V \mid H).$$

Equivalently, $\text{Req}_a(V) \not\leq \text{End}_a(V)$, while

$$\text{Req}_a(V) \leq \text{End}_a(V) \vee \text{Host}_a(H).$$

The viral replication cycle closes only in the coupled virus–host system.

14. Axiomatized conventional computers

Let C be an ordinary contemporary computer. Several signatures are available. For the restricted signature of instruction execution, one may have $\text{Cl}_{\Sigma_{\text{exec}}}(C)$: a running machine can carry out many of the operations required for continued execution using its own hardware.

Enlarge the signature to repair, chip fabrication, component replacement, assembly, and physical reproduction. For an ordinary computer, $\neg \text{Cl}_{\Sigma_{\text{phys}}}(C)$. There is instead an industrial environment I such that, for the relevant modes,

$$\text{Req}_a(C) \leq \text{End}_a(C) \vee \text{Ind}_a(I),$$

so that $\text{Cl}_a(C \mid I)$ may hold while $\text{Cl}_a(C)$ does not.

The causal picture is then approximately $I \longrightarrow C$, where I contains semiconductor fabrication, mining, refining, electricity generation, human maintenance, transport, and manufacturing infrastructure.

15. Cell/computer separation

Theorem 15.1 Cell and computer

Let B be an axiomatized cellular organism and C an ordinary computer. Suppose some mode a satisfies $\text{Cl}_a(B)$ and $\neg \text{Cl}_a(C)$. Then $B \not\approx C$ as Σ -systems.

Proof.

An equivalence $B \approx C$ would imply $\text{Cl}_a(B)$ if and only if $\text{Cl}_a(C)$, by Theorem 7.1.

16. Virus/cell separation

Theorem 16.1 Virus and cell

Suppose B is autonomous in a mode a , so $\text{Cl}_a(B)$, while a virus V is strictly host-dependent in that mode: $\neg \text{Cl}_a(V)$, though $\text{Cl}_a(V \mid H)$ for some host. Then $V \not\approx B$.

Proof.

Immediate from Theorem 7.1. Relative closure of V with H is a property of a different structured system, not of V alone.

A virus may be biologically adjacent to life without inhabiting the same full subcategory as autonomously organized cells.

17. The virus/computer analogy

There is nevertheless a legitimate abstraction under which viruses and programs resemble one another. Let Sys_Σ be the category of structured physical systems, and let

$$U : \text{Sys}_\Sigma \rightarrow \text{Dep}$$

be the forgetful functor that remembers only a replicator or executable specification, a required host, and successful production after host supplementation. It forgets material typing, biochemical organization, evolutionary lineage, boundary production, thermodynamic organization, and most of the physical causal structure.

Under U one may have $U(V) \approx U(P)$ for a virus V and a program P . Both have the shape “replicator plus host yields copies”:

$$V + H \longrightarrow V^{\sqcup n}, \quad P + M \longrightarrow P^{\sqcup n}.$$

18. Non-conservativity of the analogy

Proposition 18.1 The dependency functor need not be conservative

The functor $U : \text{Sys}_\Sigma \rightarrow \text{Dep}$ need not reflect equivalences. That is, $U(X) \approx U(Y)$ does not imply $X \approx Y$.

Proof.

Choose X and Y with $U(X) \approx U(Y)$ but $\text{Supp}(X) \not\approx \text{Supp}(Y)$. Theorem 3.1 forbids $X \approx Y$. Since U forgets material support, it may still identify their dependency skeletons.

Corollary 18.2 Analogy is not equivalence

The statement that viruses are like computer programs can be correct after a sufficiently coarse forgetful functor, without entailing that a virus is equivalent to a computer as a physical system, and still less that a computer is equivalent to a cell.

19. The original argument, categorically

Let B be a bacterium, V a virus, and C a computer. At the coarse dependency level one may have $U(V) \approx U(C)$. In the richer physical category, $V \not\approx C$ as soon as an invariant differs, and $B \not\approx V$ when autonomous closure differs. The inference

$$U(V) \approx U(C) \implies B \approx C$$

is invalid. Equality after U is equality in the quotient semantics U induces. It says nothing about the domain unless U is conservative. Since U is not conservative, the inference fails.

20. The combined invariant

Define

$$I(X) = (\text{Supp}(X), \{\text{Cl}_a(X)\}_{a \in \Sigma}).$$

This records material support together with the closure profile.

Theorem 20.1 The combined invariant is preserved

If $X \simeq Y$ as fully structured physical systems, then $I(X) = I(Y)$.

Proof.

Material support is invariant by Theorem 3.1. Each closure component is invariant by Theorem 7.1.

Corollary 20.2 Two independent obstructions

If $\text{Supp}(X) \neq \text{Supp}(Y)$, or if $\text{Cl}_a(X)$ fails to agree with $\text{Cl}_a(Y)$ for some $a \in \Sigma$, then $X \neq Y$.

There is a substrate obstruction, and there is an organizational-closure obstruction. Neither is a restatement of the other.

21. Substrate and organization are independent

The two invariants should not be collapsed. There can exist systems X and Y with $\text{Supp}(X) \neq \text{Supp}(Y)$ while both are Σ -closed. A hypothetical self-maintaining artificial organism could instantiate a different substrate and still satisfy the same closure axioms. Conversely, two systems built from similar materials may differ in organizational closure.

Neither sameness of material nor sameness of computation is sufficient for equivalence. The invariant that matters is structured organization.

22. Simulation is weaker than equivalence

Suppose a semantics or simulation functor $S : \text{PhysSys} \rightarrow \text{Comp}$ sends a biological system B to a computational representation $S(B)$. Even a perfect reproduction of some state-transition structure is only a morphism, or an equivalence, at the semantic level. One cannot infer $B \simeq S(B)$. The two need not even lie in the same category.

If S forgets $\text{Supp}(B)$ or $\text{Cl}_\Sigma(B)$, then S fails to be conservative for precisely the structure at issue.

Simulation of dynamics does not imply equivalence of physical organization.

Categorically, this is the difference between preservation of selected observables and equivalence of structured objects.

23. Open-system category theory

The framework is compatible with compositional treatments of open physical and chemical systems. Baez and Pollard model open reaction networks as morphisms, with composition given by joining outputs of one system to inputs of another.

Supplementation $X \mid H$ is a higher-categorical analogue of that composition.

The additional structure here is the separation of generic material flows from organizational enabling processes. Only the latter enter the closure invariant.

24. Relation to Rosen

Rosen's (M, R) -systems motivate the same structural separation. A system may stay open to material flow while requiring that its efficient causes be generated inside the organizational network. Later analyses call this closure to efficient causation: the catalysts or processors the system requires are themselves produced within the system.

The inequality $\text{Req}_a(X) \leq \text{End}_a(X)$ is an (∞) -topos-theoretic abstraction of that relation. It does not adopt Rosen's stronger, and controversial, claims about computability. The separation theorem uses only invariance of closure. It does not claim that organisms are noncomputable.

25. Relation to autopoiesis

Maturana's formulation requires that an autopoietic system recursively produce the components realizing its own network of production, and its boundary. The present formalism encodes this by including modes a_{boundary} and $a_{\text{production}}$ and requiring both $\text{Cl}_{a_{\text{boundary}}}(X)$ and $\text{Cl}_{a_{\text{production}}}(X)$.

A categorical formalization of autopoiesis is still an active research problem, not a settled theorem. The construction here is an axiomatic formalization, not a claim to be the unique mathematics of life.

26. Main separation theorem

Theorem 26.1 Organism, virus, and computer

Let B , V , and C in $\text{Sys}_{K,\Sigma}(E)$ represent a cellular organism, a virus, and a conventional computer. Assume:

- Cellular closure: $\text{Cl}_{\Sigma_{\text{org}}}(B)$.
- Viral host dependence: for some $a \in \Sigma_{\text{org}}$, $\neg \text{Cl}_a(V)$, but $\text{Cl}_a(V \mid H)$ for a host H .
- Computer industrial dependence: for some $b \in \Sigma_{\text{org}}$, $\neg \text{Cl}_b(C)$, but $\text{Cl}_b(C \mid I)$ for an industrial environment I .

Then $B \not\approx V$ and $B \not\approx C$. If in addition $\text{Supp}(V) \not\approx \text{Supp}(C)$, then $V \not\approx C$ as K -typed systems, even when a dependency functor satisfies $U(V) \approx U(C)$.

Proof.

The first two non-equivalences are Theorem 7.1: B is closed in a mode where V is not, and in a mode where C is not. The third is Theorem 3.1. The possible equivalence $U(V) \approx U(C)$ does not affect the conclusion, because U is not conservative.

27. Logical form of the conclusion

The invariant takes different values on the three systems:

$$\begin{array}{ccc} B & V & C \\ \downarrow I & \downarrow I & \downarrow I \\ I(B) & I(V) & I(C), \end{array}$$

with $I(B) \neq I(V)$ and $I(B) \neq I(C)$. A coarse functor may still satisfy $U(V) \approx U(C)$.

$V \not\approx C$

is compatible with a valid analogy $V \sim C$ in a quotient semantics. More importantly, whenever autonomous organizational closure is among the preserved structures,

$$B \not\sim V \quad \text{and} \quad B \not\sim C.$$

28. Final categorical formulation

There is an (∞) -category $Sys_{K,\Sigma}(E)$ of physically typed, organizationally structured systems, and full subcategories $Aut_{\Sigma}(E)$, $HostDep_{\Sigma}(E)$, and $ExtMaint_{\Sigma}(E)$, cut out respectively by

$$\begin{aligned} & Cl_{\Sigma}(X), \\ & \neg Cl_{\Sigma}(X) \wedge \exists H. Cl_{\Sigma}(X \mid H), \\ & \neg Cl_{\Sigma}(X) \wedge \exists I. Cl_{\Sigma}(X \mid I). \end{aligned}$$

The second and third predicates have the same logical shape. What differs is the supplement they quantify over — host machinery, or an external maintenance ecology — together with every other invariant the ambient category retains. At a biological scale one may model $B \in Aut_{\Sigma}$, $V \in HostDep_{\Sigma}$, and $C \in ExtMaint_{\Sigma}$. There is then no equivalence $B \simeq V$ or $B \simeq C$ preserving the full (K, Σ) -structure.

Computational equivalence is strictly coarser than organizational equivalence, which is strictly coarser than equivalence as physically typed systems.

A computer, a virus, and a bacterium become indistinguishable only after one successively forgets the structures that distinguish them. The mathematical question is never simply whether both can be represented computationally.

Which functor is being used, what structure does it forget, and does it reflect equivalences?

Without an affirmative answer to the last question, sameness of representation is not sameness of system. Essay II lifts the image inequality to operadic generation. Essay III requires that generation be thermodynamically feasible and that a viability kernel be inhabited.

References

Jacob Lurie, **Higher Topos Theory**. Princeton University Press, 2009. Chapters 6–7: (∞) -topoi, effective epimorphisms, truncation, and image factorizations.

Louis Martini and Sebastian Wolf, **Internal Higher Topos Theory**. The theory of topoi internal to an arbitrary (∞) -topos.

Humberto R. Maturana, “The Organization of the Living: A Theory of the Living Organization.” Autopoietic organization as recursively self-producing organization.

Robert Rosen, **Life Itself: A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life**. Columbia University Press, 1991. Relational biology, machines, and organizational causation.

María Luz Cárdenas, Juan-Carlos Letelier, Claudio Gutierrez, Athel Cornish-Bowden, and Jorge Soto-Andrade, “Closure to efficient causation, computability and artificial life.” **Journal of Theoretical Biology** 263 (2010), 79–92.

John C. Baez and Blake S. Pollard, “A compositional framework for reaction networks.” **Reviews in Mathematical Physics** 29 (2017), 1750028.

Tyler B. Jacobson, Melanie M. Callaghan, and Daniel Amador-Noguez, “Hostile takeover: how viruses reprogram prokaryotic metabolism.” Dependence of viral reproduction on host cellular machinery.

Peter D. Nagy and Judit Pogany, “The dependence of viral RNA replication on co-opted host factors.” **Nature Reviews Microbiology** 10 (2012), 137–149.

Operadic Autopoiesis in an (∞) -Topos

Abstract

Let E be an (∞) -topos containing an internal symmetric monoidal (∞) -category $P^\otimes$ of open physical processes, and let $O^\otimes$ be the associated internal colored (∞) -operad. Colors are interfaces, resources, and organizational roles. Multimorphisms are composable physical operations.

For a system X , write $E_X^\otimes \hookrightarrow O^\otimes$ for the endogenous process operad, $R_X^\otimes \hookrightarrow O^\otimes$ for the required organizational operad, and $G_X^\otimes$ for the least fixed point of an accessible production operator applied to E_X . Operadic autopoiesis is the inclusion

$$R_X^\otimes \subseteq G_X^\otimes.$$

Every organizational operation required for the continuation of X is realizable by a recursively self-generated network internal to X .

Interaction across a common interface operad $I^\otimes$ is the homotopy pushout $E_X^\otimes \sqcup_I E_H^\otimes$. The pushout contains mixed composites, not merely the union of the two operads. A host-dependent replicator then satisfies $R_V \not\subseteq G_V$ but $R_V \subseteq G_{V \star H}$. A cell satisfies $R_B \subseteq G_B$. An ordinary computer, once fabrication, repair, replacement, and physical reproduction are required, satisfies $R_C \not\subseteq G_C$.

These properties are invariant under equivalence, local-to-global along effective epimorphisms, and separate autonomous systems from externally dependent ones. A non-conservativity theorem explains why equivalence of dependency patterns, or of computational observations, does not yield equivalence of physical systems.

1. Ambient (∞) -topos

Let E be an (∞) -topos, interpreted as a generalized space of physical contexts: spacetime regions, experimental conditions, environments, or organizational scales. Systems and processes may vary locally, and they satisfy descent. Internal higher categories in an (∞) -topos may be studied as suitable sheaves of (∞) -categories. The internal theory required below is available in that setting.

2. Internal (∞) -categories

Write $\text{Cat}_\infty(E)$ for the (∞) -category of internal (∞) -categories in E . Concretely, an internal (∞) -category may be presented as a complete Segal object $C : \Delta^{\text{op}} \rightarrow E$. Then C_0 is the object of objects, C_1 the object of morphisms, and C_n the object of coherently composable n -fold strings. The Segal equivalences read

$$C_n \simeq C_1 \times_{C_0} \cdots \times_{C_0} C_1,$$

and completeness identifies equivalences internally.

3. Internal symmetric monoidal (∞) -categories

Definition 3.1 Internal symmetric monoidal (∞) -category

An internal symmetric monoidal (∞) -category is a commutative algebra

$$P^{\otimes} \in \text{CAlg}(\text{Cat}_{\infty}(E)).$$

The multiplication $\otimes : P \times P \rightarrow P$ is parallel composition. The categorical composition $\circ$ is sequential composition. The symmetric monoidal coherences hold internally in E .

4. Open physical processes

Interpret $P^{\otimes}$ as the internal symmetric monoidal (∞) -category of open physical processes. Objects are interfaces: chemical species, membranes, material stocks, energy carriers, molecular machines, electrical ports, mechanical ports. A morphism $p : A \rightarrow B$ is an open process taking the input interface A to the output interface B .

Definition 4.1 Open realization

An open process may be represented by a cospan-shaped diagram $L(A) \rightarrow X \leftarrow L(B)$, where X is the system performing the transformation and $L(A)$, $L(B)$ are its exposed interfaces. Sequential composition is gluing along a common interface, $X \sqcup_{L(B)} Y$. Parallel composition is $X \otimes Y$.

This is the higher-categorical analogue of the structured cospan used for open reaction networks, electrical circuits, and Petri nets. Ordinary pushouts are replaced by homotopy-coherent pushouts.

5. The process (∞) -operad

A symmetric monoidal (∞) -category determines an (∞) -operad. In Lurie's formalism an (∞) -operad is a map $O^{\otimes} \rightarrow N(\text{Fin}_{\star})$ with inert lifts and Segal conditions. We use the internal analogue.

Definition 5.1 Process operad

Let $O^{\otimes}$ be the internal (∞) -operad underlying $P^{\otimes}$. For colors $x_1, \dots, x_n, y$ there is an object

$$\text{Mul}_O(x_1, \dots, x_n; y) \in E$$

of physical n -ary operations $(x_1, \dots, x_n) \rightarrow y$. Operadic substitution supplies

$$\text{Mul}(y_1, \dots, y_n; z) \times \prod_i \text{Mul}(x_{i1}, \dots, x_{ik_i}; y_i) \longrightarrow \text{Mul}(x_{11}, \dots; z).$$

A physical causal network is a tree of composable operations, not merely a sequence of maps.

6. Internal suboperads

Definition 6.1 Suboperad

A suboperad $A^{\otimes} \hookrightarrow O^{\otimes}$ consists of subobjects

$$\text{Mul}_A(\tilde{x}; y) \hookrightarrow \text{Mul}_O(\tilde{x}; y)$$

for every profile $(\tilde{x}; y)$, stable under units, permutations, inert operations, operadic substitution, and restriction in E . Write $\text{SubOp}(O)$ for the poset of suboperads, ordered by inclusion. We write both $A \leq B$ and $A \subseteq B$ for that order.

Lemma 6.2 Intersections of suboperads

An arbitrary intersection of suboperads of $O^\otimes$ is a suboperad.

Proof.

Intersection is computed profilewise,

$$\text{Mul}_{\bigcap_i A_i}(\mathbf{x}; \mathbf{y}) = \bigcap_i \text{Mul}_{A_i}(\mathbf{x}; \mathbf{y}).$$

Each factor contains the operadic units and is stable under permutation and substitution, so the intersection is as well.

Definition 6.3 Generated suboperad

For a collection S of operations of O , the generated suboperad $\langle S \rangle_O$ is the intersection of all suboperads containing S . It is the smallest suboperad containing S .

7. Systems as bounded operadic realizers

Definition 7.1 Structured physical system

A structured physical system is a tuple

$$X = (|X|, \partial X, E_X^\otimes, R_X^\otimes, \Phi_X).$$

Here $|X| \in E$ is the physical carrier, ∂X its interaction boundary, $E_X^\otimes \hookrightarrow O^\otimes$ its primitive endogenous process operad, $R_X^\otimes \hookrightarrow O^\otimes$ its organizational requirement operad, and Φ_X its recursive production operator.

8. Endogenous processes

Definition 8.1 Endogenous operation

An operation $f \in \text{Mul}_O(\mathbf{x}; \mathbf{y})$ is endogenous to X when a physical realizer of f lies inside the causal boundary of X . The endogenous suboperad $E_X^\otimes$ contains every operation realizable from structures belonging to X , and it is closed under operadic composition: if $f, g_1, \dots, g_n \in E_X$, then $f \circ (g_1, \dots, g_n) \in E_X$.

9. Organizational requirements

Endogeneity is not enough. A rock has endogenous physical processes. The question is whether the processes required for the organization to persist are generated by that organization.

Definition 9.1 Requirement operad

The requirement operad $R_X^\otimes$ is the suboperad generated by those organizational operations whose continued realization is required for X to remain in the chosen viability class. Generators may include boundary maintenance, repair, catalysis, translation, fabrication, replication, regulation, and energy transduction. The signature is selectable. Different theories of life correspond, in part, to different choices of $R_X^\otimes$.

10. Why one-step generation is not enough

Suppose an organism currently possesses a ribosome. Translation is then an endogenous operation. That does not say whether the system can produce or repair the machinery that realizes translation. Closure has to be recursive, not a snapshot of the operations available at one moment.

11. Production doctrine

Definition 11.1 Production doctrine

A production doctrine for X is a monotone endomap $\Phi_X : \text{SubOp}(O) \rightarrow \text{SubOp}(O)$. The value $\Phi_X(A)$ contains those organizational operations whose physical realizers X can construct, maintain, regenerate, or repair using operations from A , together with the primitive endogenous capabilities and the permitted matter–energy inputs. Monotonicity means $A \leq B$ implies $\Phi_X(A) \leq \Phi_X(B)$.

Assumption 11.2 Accessibility

The operator Φ_X is accessible, and inflationary once the primitive endogenous operations have been adjoined. For a sufficiently large regular ordinal λ , Φ_X preserves the relevant λ -filtered colimits. Recursive generation may therefore be computed by transfinite iteration.

12. Recursive endogenous closure

Set $G_X^{(0)} := E_X$. At a successor ordinal,

$$G_X^{(\alpha+1)} := \Phi_X(G_X^{(\alpha)}).$$

At a limit ordinal β ,

$$G_X^{(\beta)} := \bigcup_{\alpha < \beta} G_X^{(\alpha)}.$$

For sufficiently large λ the iteration stabilizes. Write $G_X := G_X^{(\lambda)}$, and also $G_X = \mu\Phi_X$ for this least fixed point.

Proposition 12.1 Fixed point

The operad G_X is a fixed point of Φ_X : $\Phi_X(G_X) \approx G_X$.

Proof.

By accessibility,

$$\Phi_X(\text{colim}_{\alpha < \lambda} G_X^{(\alpha)}) \approx \text{colim}_{\alpha < \lambda} \Phi_X(G_X^{(\alpha)}).$$

The right-hand side is the colimit of the successor stages $G_X^{(\alpha+1)}$. Shifting the ordinal index does not change the colimit, so $\Phi_X(G_X) \approx G_X$.

Proposition 12.2 Least fixed point

The operad G_X is the least Φ_X -closed suboperad containing E_X .

Proof.

Let A contain E_X and satisfy $\Phi_X(A) \leq A$. Transfinite induction gives $G_X^{(\alpha)} \leq A$ for every α . Successor stages use monotonicity. Limit stages use that A is itself a suboperad containing every earlier stage, hence contains the suboperad they generate. Therefore $G_X \leq A$.

13. Operadic organizational closure

Definition 13.1 Operadic autopoiesis

The system X is organizationally closed, written $\text{Aut}(X)$, when

$$R_X^\otimes \subseteq G_X^\otimes.$$

Every operation organizationally required by X lies in the recursively generated endogenous operad.

14. Interpretation

The inclusion $R_X \subseteq G_X$ does not say that X is materially isolated. Environmental resource morphisms $M_{\text{env}} \rightarrow X$ and $E_{\text{env}} \rightarrow X$ are allowed. Matter and energy may flow through the system. Closure concerns the generation of organizational constraints and of productive machinery.

Organizational closure is not thermodynamic closure.

This matches the classical autopoietic idea: a living system recursively produces the network and the boundary that constitute it.

15. Equivalence of structured systems

Definition 15.1 Equivalence

An equivalence $X \simeq Y$ of structured systems consists of compatible equivalences $|X| \simeq |Y|$, $\partial X \simeq \partial Y$, $E_X \simeq E_Y$, and $R_X \simeq R_Y$, conjugating the production operators:

$$F_* \circ \Phi_X \simeq \Phi_Y \circ F_*.$$

16. Fixed-point invariance

Theorem 16.1 Equivalences preserve generative closure

If $X \simeq Y$ is an equivalence of structured systems, then $F_*(G_X) \simeq G_Y$.

Proof.

At stage zero, $F_*(E_X) \simeq E_Y$, so $F_*(G_X^{(0)}) \simeq G_Y^{(0)}$. If the claim holds at stage α , conjugacy of the production operators gives

$$F_* \Phi_X(G_X^{(\alpha)}) \simeq \Phi_Y F_*(G_X^{(\alpha)}) \simeq \Phi_Y(G_Y^{(\alpha)}),$$

which is the successor stage of Y . At limit ordinals, F_* preserves the generated colimits in question. By transfinite induction the stages agree, and therefore so do the stable values.

17. Autopoiesis is invariant

Theorem 17.1 Invariance of operadic autopoiesis

If $X \simeq Y$, then $\text{Aut}(X)$ if and only if $\text{Aut}(Y)$.

Proof.

Suppose $R_X \subseteq G_X$. Applying F_* and using preservation of requirements together with Theorem 16.1 yields $R_Y \subseteq G_Y$. The converse uses F^{-1} .

Corollary 17.2 Separation by autopoiesis

If $\text{Aut}(X)$ and $\neg\text{Aut}(Y)$, then $X \neq Y$.

This already separates an autonomously closed cell from an externally maintained machine, inside the present model.

18. Interaction interfaces

Let X and H interact through an interface operad $I^\otimes$, with operad morphisms $i_X : I^\otimes \rightarrow E_X^\otimes$ and $i_H : I^\otimes \rightarrow E_H^\otimes$. Interface colors may include membrane receptors, ribosomal binding sites, ATP, nucleotides, polymerases, and transport machinery.

19. Operadic coupling

Definition 19.1 Coupled operad

The raw coupled operad is the homotopy pushout

$$E_X \sqcup_I E_H.$$

Whenever the two endogenous realizations agree on the common interface, there is a canonical morphism

$$j : E_X \sqcup_I E_H \longrightarrow O.$$

Write $E_{X \star H} := \langle \text{im}(j) \rangle_O$ for the suboperad it generates.

20. Why the pushout is larger than the union

The pushout contains more than $E_X \cup E_H$. It contains operations obtained by composing an operation from one system with operations from the other. Schematically, $f_X \circ (g_H, h_X, k_H)$ is legal whenever the colors match. The coupled system has mixed causal pathways that neither system has alone.

21. Mixed trees

Lemma 21.1 Mixed-tree lemma

Every finite operadic tree whose vertices are labeled by operations from E_X or from E_H , and whose cross-system internal edges have colors in the common interface I , determines an operation of $E_{X \star H}$.

Proof.

Induct on height. A tree of height zero is a single operation of X or of H , hence lies in the image of the pushout. If the claim holds up to height n , and T has height $n + 1$, each immediate subtree of the root represents, by induction, an operation of the pushout-generated operad. The root operation lies in E_X or in E_H . Matching edge colors make the operadic substitution defined, and closure under substitution produces the operation corresponding to T .

22. Emergent causal capacity

Corollary 22.1 Coupling creates operations

An operation r may satisfy $r \notin E_X$ and $r \notin E_H$ while $r \in E_{X \star H}$.

Proof.

The operation r may be a composite tree with some vertices supplied by X and others by H . Lemma 21.1 places that tree in $E_{X \star H}$.

Coupled capability is not the union of the isolated capabilities.

This is the structure needed for viral use of host-cell machinery.

23. Coupled production

The coupled system has a production doctrine $\Phi_{X \star H}$ whose primitive stage contains $E_{X \star H}$. Its recursively generated operad is $G_{X \star H} := \mu\Phi_{X \star H}$.

24. Relative closure

Definition 24.1 Closure relative to a host

The system X is organizationally closed relative to H , written $\text{Aut}(X \mid H)$, when $R_X \subseteq G_{X \star H}$.

25. Strict host dependence

Definition 25.1 Strict dependence

The system X is strictly dependent on H when $\neg\text{Aut}(X)$ but $\text{Aut}(X \mid H)$. Equivalently, $R_X \not\subseteq G_X$ while $R_X \subseteq G_{X \star H}$.

26. The virus theorem

Let V be a virus and H a susceptible host cell. Viral replication depends on host translation, and many viruses co-opt host proteins, membranes, metabolism, and other cellular processes for genome replication and assembly. The modeling hypothesis is

$$R_V \not\subseteq G_V, \quad R_V \subseteq G_{V \star H}.$$

Theorem 26.1 Viral host-dependence

Under these hypotheses, V is a strictly host-dependent organizational system.

Proof.

This is Definition 25.1. The content is in the structure that instantiates the predicates: $G_{V \star H}$ contains mixed virus–host operations produced by the operadic pushout, which G_V does not.

27. The cell

Let B be a bacterial cell, and let R_B be generated by membrane maintenance, repair, translation, replication, metabolism, regulation, and the other modes treated as essential. The biological-autonomy hypothesis is $R_B \subseteq G_B$. This is an axiomatization, not a theorem of biology. Inside the theory it says that the cell’s required organizational machinery lies in its recursively regenerative operadic closure.

28. Virus and cell

Theorem 28.1 Virus–cell separation

If $\text{Aut}(B)$ and $\neg\text{Aut}(V)$, then $B \not\approx V$ as structured physical systems.

Proof.

Operadic autopoiesis is invariant under equivalence by Theorem 17.1. An equivalence $B \approx V$ would force $\text{Aut}(B)$ if and only if $\text{Aut}(V)$.

29. The computer

Let C be a conventional computer. On a restricted operational signature — instruction execution, memory access, arithmetic — the machine may have substantial endogenous closure. Take instead the physically complete requirement operad R_C^{phys} generated by chip fabrication, replacement, repair, structural reproduction, power infrastructure, and assembly. For an ordinary present-day computer,

$$R_C^{\text{phys}} \not\subseteq G_C,$$

while an industrial infrastructure I supplies the missing operations, $R_C^{\text{phys}} \subseteq G_C \star I$. Under the physical requirement signature, $\neg\text{Aut}(C)$.

30. Cell and computer

Theorem 30.1 Cell–computer separation

If $\text{Aut}(B)$ and $\neg\text{Aut}(C)$, then $B \not\approx C$ as operadically structured physical systems.

Proof.

Theorem 17.1.

31. Stronger than a list of parts

A purely material argument says $\text{Supp}(B) \neq \text{Supp}(C)$. The operadic argument says $G_B \neq G_C$ as organizational process algebras. The first is a difference of objects. The second is a difference of the higher algebra of transformations among objects.

The distinction is not merely different parts. It is different recursively generated causal algebras.

32. Material typing

The substrate invariant remains available. Let $K \in E$ be the object of material component types, and let $\tau_X : C_X \rightarrow K$ be the typing of the components of X . Set $\text{Supp}(X) := \text{im}(\tau_X)$.

Proposition 32.1 Support is invariant

An equivalence that preserves material typing induces $\text{Supp}(X) \approx \text{Supp}(Y)$.

Proof.

As in the typed setting: if $\tau_X \approx \tau_Y \circ f$ with f an equivalence, the images agree.

Material support is an invariant independent of the closure truth value.

33. The full physical invariant

Define

$$J(X) = (\text{Supp}(X), E_X, R_X, \Phi_X, G_X).$$

Theorem 33.1 Structured equivalence preserves J

A structured equivalence $X \simeq Y$ induces $J(X) \simeq J(Y)$. Any difference among these components obstructs physical equivalence.

Proof.

Support is Proposition 32.1. The operads E and R , and the operator Φ , are preserved by Definition 15.1. The generated operad G is preserved by Theorem 16.1.

34. Why a virus and a program can look alike

Let V be a virus and P a self-copying program. There is a coarse dependency pattern $V + H \rightarrow V^{\sqcup n}$ and $P + M \rightarrow P^{\sqcup n}$. Define a forgetful functor $Q : \text{PhysSys} \rightarrow \text{Dep}$ that remembers only specification, host, and replication. It is entirely possible that $Q(V) \simeq Q(P)$.

35. Non-conservativity

Theorem 35.1 Forgetting support or the operad destroys conservativity

If Q forgets either material support or the endogenous operad, then Q is not conservative in general. Thus $Q(X) \simeq Q(Y)$ does not imply $X \simeq Y$.

Proof.

Choose X and Y with the same dependency diagram and with $\text{Supp}(X) \neq \text{Supp}(Y)$. Then $Q(X) \simeq Q(Y)$. If $X \simeq Y$ held in the full category, Proposition 32.1 would force the supports to agree.

36. The virus/program analogy

Therefore $Q(V) \simeq Q(P)$ may hold even though $V \neq P$. The analogy lives in a quotient of the structural theory. It is invalid to infer $V \simeq P$ from $Q(V) \simeq Q(P)$, and still more invalid to infer $B \simeq P$ for a bacterium B .

37. Localization

The same phenomenon can be phrased as a localization. Let $L : E \rightarrow E_{\text{comp}}$ be an accessible left-exact localization, read as a computational observational modality. Interpret LX as what is observable about X through the chosen computational semantics.

Definition 37.1 Computational equivalence

Systems X and Y are computationally equivalent when $LX \simeq LY$.

Proposition 37.2 Non-conservative localizations identify non-equivalent systems

If L is not conservative, there exist $X \neq Y$ with $LX \simeq LY$.

Proof.

Failure of conservativity is exactly failure to reflect equivalences.

38. Simulation

Suppose a bacterium and a computer simulation satisfy $L(B) \simeq L(C)$. All that follows is equality after computational localization. One cannot conclude $B \simeq C$ unless L reflects equivalences on the relevant class of systems.

Equality after computational localization does not imply equality in the ambient category.

39. Locality

Let $u : U \rightarrowtail 1$ be an effective epimorphism, and assume the production doctrine is geometric:

$$u^* \Phi_X(A) \simeq \Phi_{u^* X}(u^* A).$$

Inverse-image functors preserve colimits, so transfinite generation commutes with pullback: $u^* G_X \simeq G_{u^* X}$.

40. Descent for closure

Theorem 40.1 Closure descends

If $u^* R_X \subseteq u^* G_X$ for an effective epimorphism $u : U \rightarrowtail 1$, then $R_X \subseteq G_X$.

Proof.

Suboperad inclusion is profilewise inclusion of subobjects. For each multimorphism profile, $u^* R(\mathfrak{x}; y) \leq u^* G(\mathfrak{x}; y)$. Pullback along an effective epimorphism reflects subobject inclusion, so $R(\mathfrak{x}; y) \leq G(\mathfrak{x}; y)$ for every profile.

41. Host coupling is not a change of context

Corollary 41.1 Observation does not create autonomy

If $\neg \text{Aut}(V)$, effective-epimorphic base change alone cannot produce $\text{Aut}(u^* V)$.

Proof.

If $\text{Aut}(u^* V)$ held, Theorem 40.1 would give $\text{Aut}(V)$.

Host infection is not a change of context. It is operadic enlargement by coupling.

The host has to contribute operations. That is the point of the pushout.

42. Three loci

Suppose structured systems form an internal moduli object, or a stack, Sys over E . The autonomous locus, the merely dependent locus, and the host-dependent locus are the internally definable subobjects

$$\begin{aligned} Aut &= \{X \mid R_X \subseteq G_X\}, \\ Dep &= \{X \mid R_X \not\subseteq G_X\}, \\ HostDep &= \{X \mid \neg Aut(X) \wedge \exists H. Aut(X \mid H)\}. \end{aligned}$$

Thus $Aut \hookrightarrow Sys$ and $HostDep \hookrightarrow Sys$ are internal loci, not external labels pasted onto the mathematics.

43. An important non-theorem

It does not follow that $HostDep$ is the class of viruses. Computers can be externally dependent. Parasitic organisms can be externally dependent. Organelles can be externally dependent. Dependence is an invariant. It is not a definition of “virus.” Further invariants are required.

44. A stratified invariant

Enrich the invariant by hereditary organization, lineage, and self-produced boundary:

$$K(X) = (Supp(X), G_X, R_X, Her(X), Lin(X), Bound(X)).$$

Cells, viruses, and computers may then occupy distinct strata even where one closure coordinate agrees.

45. One invariant proves little

Suppose $Aut(V)$ and $Aut(C)$ are both false. It does not follow that $V \simeq C$. Both objects merely lie in the same fiber of $Aut : Sys \rightarrow \Omega$. A fiber can contain many non-equivalent objects. The locus $Aut^{-1}(\perp)$ is, potentially, enormous.

46. A hierarchy of forgetful semantics

There is a chain of forgetful functors

$$PhysSys \xrightarrow{F_1} OrgSys \xrightarrow{F_2} DepSys \xrightarrow{F_3} CompSem.$$

The first forgets detailed substrate, the second forgets most organizational structure, and the third retains only computationally relevant behavior. One may have

$$F_3 F_2 F_1(B) \simeq F_3 F_2 F_1(C)$$

while $B \not\simeq C$.

47. Forgetfulness in general

Theorem 47.1 Reflection is conservativity

Let $F : C \rightarrow D$ be any functor. The inference $F(X) \simeq F(Y) \implies X \simeq Y$, for all X and Y , holds if and only if F is conservative.

Proof.

A functor is conservative precisely when it reflects equivalences.

Corollary 47.2 The missing step

Any argument that identifies biological and computational systems through a functor forgetting metabolic, material, organizational, boundary, or reproductive structure must separately establish conservativity. Without that step the conclusion does not follow.

48. Main separation theorem

Theorem 48.1 Operadic separation

Let B , V , and C be a cellular organism, a virus, and a conventional computer in $\text{Sys}_{E,O}$. Assume $R_B \subseteq G_B$; $R_V \not\subseteq G_V$; $R_V \subseteq G_{V \star H}$ for some host H ; and $R_C \not\subseteq G_C$. Assume also that at least one of $\text{Supp}(V)$, G_V , $\text{Her}(V)$, and $\text{Lin}(V)$ differs from the corresponding invariant of C . Then

$$B \neq V, \quad B \neq C, \quad V \neq C$$

in the fully structured category.

Proof.

The first two non-equivalences are Theorem 17.1, since B is organizationally closed and V and C are not. For the third, an equivalence would preserve every component of K . By hypothesis at least one component differs.

49. What the equations say about a bacterium

The bacterium is not distinguished merely by having proteins rather than transistors. The stronger property in the model is $R_B \subseteq \mu\Phi_B$: the operations required to regenerate the organism lie in the least recursively closed operadic algebra generated by processes internal to the organism.

Requirements are contained in recursive endogenous generative closure.

50. What the equations say about a virus

For a virus, $R_V \not\subseteq \mu\Phi_V$, but $R_V \subseteq \mu\Phi_{V \star H}$. Organizational closure of the virus is a property of the coupled virus–host system. The pushout $E_V \sqcup_I E_H$ is what supplies the missing mixed operations.

51. What the equations say about a computer

For a computer, $R_C^{\text{phys}} \not\subseteq \mu\Phi_C$, while one may have $R_C^{\text{phys}} \subseteq \mu\Phi_{C \star I}$, where I is an industrial-technological system. The analogue of the viral host is not another computer. It is the industrial ecology that manufactures and maintains computers.

52. A sharper analogy, still not an equivalence

In one coordinate, the pair (virus, cell) is analogous to the pair (computer, industrial civilization). Both are externally completed systems, $V \star H$ and $C \star I$. The analogy still does not give $V \approx C$. The coupled operads, material types, interfaces, lineages, and organizational constraints remain different.

53. Higher homotopies

An (∞) -operad, rather than an ordinary operad, matters when a biological operation has many realizations. If r can be implemented by processes f and g , one need not impose $f = g$. There may be a path $f \approx g$, and paths between paths. The multimorphism object $\text{Mul}_O(\mathbf{x}; \mathbf{y})$ retains the homotopy type of the space of realizations. Organizational roles can be invariant under coherent replacement of realizers.

54. Robustness as homotopy

A biological function may correspond not to one operation but to a connected component $[f] \in \pi_0 \text{Mul}_O(\mathcal{X}; \mathcal{Y})$. Robust organization can involve large spaces of mutually substitutable realizations. Functional degeneracy can be probed by the higher homotopy groups $\pi_k \text{Mul}_O(\mathcal{X}; \mathcal{Y})$. The group π_0 records disconnected classes of solutions. Higher groups record coherent families of substitutions. Ordinary graphs and state machines discard this structure.

55. Evolution as deformation

An organism at time t may be represented by an algebra $A_t \in \text{Alg}_O(V)$ for an internal symmetric monoidal target V . Mutation induces deformations $A_t \rightsquigarrow A_{t+1}$. Evolutionary families are paths, or more general diagrams, in $\text{Alg}_O(V)$. The evolutionary object is not merely a string of genomes. It is a family of operadic realizations.

56. Closure is scale-relative

At the scale of an individual virion, $\neg \text{Aut}(V)$. At an ecological or lineage scale one may form another object L_V containing virions, hosts, transmission, mutation, and selection. It is possible that $R_{L_V} \subseteq G_{L_V}$ for a suitably enlarged boundary. Closure depends on the scale at which the system is cut out. That is not an ambiguity in the predicate. It is a change in the object to which the predicate is applied.

57. Enlarging the boundary

Let $X \subseteq Y$ be two choices of system boundary. In general $E_X \subseteq E_Y$, and therefore $G_X \subseteq G_Y$ when the production doctrines are monotone and compatible. A system that fails autonomy at one boundary may satisfy it after the boundary is enlarged. This is not a contradiction. The object has changed.

58. Do not move the boundary in silence

The distinction between V and $V \star H$ is parallel to the distinction between C and $C \star I$. One must not slide between these boundaries without saying so. Otherwise autonomy can be manufactured by changing what counts as the system.

59. Relation to the autopoiesis problem

This operadic formalization is a proposed model, not a settled definition of life. A full categorical-thermodynamic calculus of autopoietic closure is still a research program. Recent work has explored categorical sketches of self-production and viability, with the same emphasis on structural invariants and on the gap between a formal model and a biological system.

The present construction places organizational closure specifically in internal (∞) -operadic generative closure, and places host supplementation specifically in the homotopy pushout of endogenous process operads.

60. Final formulation

For each physical system X , the endogenous operad generates a chain

$$E_X \subseteq \Phi_X(E_X) \subseteq \Phi_X^2(E_X) \subseteq \cdots,$$

whose stable value is $\mu\Phi_X$. The system is autonomous exactly when $R_X \subseteq \mu\Phi_X$. For interacting systems, the span $E_X \leftarrow I \rightarrow E_H$ produces the pushout, and therefore $G_{X \star H}$. A host-dependent system satisfies $R_X \not\subseteq G_X$ but $R_X \subseteq G_{X \star H}$.

- Cell: $R_B \subseteq G_B$
- Virus: $R_V \not\subseteq G_V$, but $R_V \subseteq G_{V \star H}$

- Ordinary computer: $R_C \not\subseteq G_C$, but $R_C \subseteq G_C \star I$

These equations do not identify viruses with computers. They say that both may be externally dependent. Their full invariants still differ, $K(V) \neq K(C)$, so $V \neq C$ in the physically structured (∞) -category even where a coarse quotient satisfies $Q(V) \simeq Q(C)$. Likewise $B \neq C$ whenever the recursively generated organizational operads differ.

Sameness of input–output behavior is coarser than sameness of dependency pattern, which is coarser than sameness of process operad, which is coarser than sameness of generative closure, which is coarser than equivalence as physical systems.

The characteristic error in identifying computers with organisms is to establish an equivalence only after one or more non-conservative forgetful functors or localizations, and then to read that localized equivalence back in the ambient category. In categorical language, $F(X) \simeq F(Y)$ is evidence for $X \simeq Y$ only when F reflects equivalences. Without conservativity, the inference is invalid. That is the obstruction.

References

Jacob Lurie, **Higher Algebra**. Chapter 2: (∞) -operads, operadic algebras, coproducts of operads, and symmetric monoidal envelopes.

Louis Martini and Sebastian Wolf, “Presentable categories internal to an (∞) -topos.” Presentable internal categories and their symmetric monoidal structure.

Louis Martini and Sebastian Wolf, **Internal Higher Topos Theory**. Higher-categorical constructions internal to an arbitrary (∞) -topos.

Louis Martini and Sebastian Wolf, “Colimits and cocompletions in internal higher category theory.” Internal colimits and Kan extensions used in the recursive and pushout constructions.

David Gepner and Rune Haugseng, “Enriched ∞ -categories via non-symmetric ∞ -operads.”

John C. Baez and Kenny Courser, “Structured cospans.” **Theory and Applications of Categories** 35 (2020), 1771–1822. Open systems glued along interfaces.

Humberto R. Maturana, “The Organization of the Living: A Theory of the Living Organization.” Biological autonomy as a recursively self-producing network.

Peter D. Nagy and Judit Pogany, “The dependence of viral RNA replication on co-opted host factors.” **Nature Reviews Microbiology** 10 (2012), 137–149.

Derek Walsh and Ian Mohr, “Viral subversion of the host protein synthesis machinery.” **Nature Reviews Microbiology** 9 (2011), 860–875.

Rodrigo Ramírez-Tagle, “Formalizing autopoiesis: toward a categorical-thermodynamic calculus of closure.” **BioSystems** (2026). The categorical formalization of autopoiesis as an open problem.

Thermodynamically Sustained Autopoiesis in an (∞) -Topos

Abstract

Let E be an (∞) -topos and $P^\otimes$ an internal symmetric monoidal (∞) -category of open physical processes, with operad $O^\otimes$. For a structured system X , let E_X be its primitive endogenous operad, R_X its requirement operad, and Φ_X its production operator, with least fixed point $G_X = \mu\Phi_X$.

Thermodynamics enters through an internal ordered commutative monoid, or quantale, Q of costs, and a lax symmetric monoidal valuation $\Theta : P^\otimes \rightarrow BQ^\otimes$. In physical models Θ may record work, exergy consumption, free-energy dissipation, entropy production, or a tuple of these. The state object S_X carries a viability subobject $V_X \hookrightarrow S_X$. Admissible dynamics determine a monotone operator Ψ_X on $\text{Sub}(S_X)$, whose greatest fixed point $V_X = \nu\Psi_X$ is the viability kernel: the states from which X can remain viable indefinitely.

Thermodynamically sustained autopoiesis is the conjunction

$$R_X \subseteq \mu\Phi_X^{\text{th}} \quad \text{and} \quad V_X \neq 0,$$

where Φ_X^{th} retains only thermodynamically feasible realizations. Life-like autonomy is the coexistence of a least fixed point of recursive production and a greatest fixed point of viable dynamics. Cells, viruses, and ordinary computers are then separated by how endogenous generation, environmental coupling, viable maintenance, and thermodynamic expenditure sit relative to one another — not by information-processing capacity alone.

1. The ambient physical (∞) -topos

Let E be an (∞) -topos containing internal objects of real quantities, among them R , $R_{\geq 0}$, and $R_{\geq 0} = [0, \infty]$. Objects and morphisms may depend on spacetime, environment, experimental context, or scale. Thermodynamic quantities are variable objects of the topos, not globally fixed scalars.

2. Open physical processes

Let $P^\otimes \in \text{CAlg}(\text{Cat}_\infty(E))$ be an internal symmetric monoidal (∞) -category. Objects are physical interfaces. A morphism $p : A \rightarrow B$ is an open process. The tensor product $p \otimes q$ is parallel composition, and $q \circ p$ is sequential composition. Open reaction networks sit in this pattern: they are morphisms, and composition joins outputs of one network to inputs of another.

3. The cost object

Definition 3.1 Internal thermodynamic quantale

Let Q be an internal commutative quantale in E . The basic model is $Q = ([0, \infty], \geq, +, 0)$, with the reversed order of Lawvere’s convention: $a \leq_Q b$ if and only if $a \geq_R b$. Addition is the monoidal operation. The point of the reversal is that enrichment records cost, and a cheaper process is a “better” morphism.

Lawvere’s observation that metric spaces are categories enriched in $([0, \infty], \geq, +, 0)$ is the cost semantics used below.

4. Thermodynamic valuation

Two quantities must be kept apart. For a concrete physical realization p , entropy production or dissipated work is an actual physical number. For an abstract operation f , one may instead ask for the greatest lower bound of that cost over all realizers. The first is an additive valuation. The second is a Lawvere cost.

Definition 4.1 Realized dissipation

A thermodynamic valuation is a lax symmetric monoidal functor $\Theta : P^{\otimes} \rightarrow BQ^{\otimes}$. For a process p , write $\Theta(p) \in Q$. Depending on the model, $\Theta(p)$ may be dissipated work $W_{\text{diss}}(p)$, consumed exergy $B(p)$, environmental temperature times entropy production $T_0 \Sigma(p)$, or a tuple of such quantities.

5. Sequential and parallel composition

For independently composed realized processes one expects additivity, $\Sigma(p \otimes q) = \Sigma(p) + \Sigma(q)$ and $\Sigma(q \circ p) = \Sigma(p) + \Sigma(q)$, up to the bookkeeping of the thermodynamic model. For minimum costs, exact additivity is too strong. What survives is the triangle inequality $c(q \circ p) \leq c(p) + c(q)$.

Proposition 5.1 Lawvere cost inequality

Define $c(f) := \inf_{p \models f} \Theta(p)$, where $p \models f$ means that the physical process p realizes the abstract operation f . Then $c(g \circ f) \leq c(f) + c(g)$.

Proof.

If $p \models f$ and $q \models g$, then $q \circ p \models g \circ f$, so $c(g \circ f) \leq \Theta(q \circ p)$. Under additive accounting, $\Theta(q \circ p) = \Theta(p) + \Theta(q)$. Taking the infimum over realizers yields the inequality. Lax additivity gives the same bound.

Minimum thermodynamic cost is an enriched geometry on the space of realizable operations.

6. Nonequilibrium free energy

For biochemistry the relevant thermodynamics is nonequilibrium thermodynamics. Open chemical reaction networks admit energy and entropy balances in which nonequilibrium Gibbs free energies act as potentials. Rao and Esposito show that the entropy production of an open network decomposes into a change of potential and the work supplied through external chemostats and nonconservative forces.

At constant environmental temperature T_0 , a schematic splitting of supplied free energy is

$$W_{\text{in}} = \Delta F + T_0 \Sigma + W_{\text{out}}, \quad \Sigma \geq 0.$$

In rates, for an open material system,

$$W_{\text{in}} = \dot{F} + J_{\text{out}}^F + T_0 \dot{\Sigma} + W_{\text{out}}.$$

The potential depends on the ensemble and on the conserved quantities. The categorical theory needs only an additive or lax-additive valuation Θ . The displays above are the intended physical reading of that valuation, not an extra axiom.

7. Entropy production

Definition 7.1 Entropy production

For each process $p : A \rightarrow B$, let $\Sigma(p) \in R_{\geq 0}$ be its entropy production. The second law is the internal proposition $\Sigma(p) \geq 0$. When the interface accounting includes every exchanged entropy, sequential composition adds: $\Sigma(q \circ p) = \Sigma(p) + \Sigma(q)$. For independent subsystems, parallel composition adds: $\Sigma(p \otimes q) = \Sigma(p) + \Sigma(q)$.

8. Exergy

Energy is too coarse a currency, because energy is conserved. What an autonomous system consumes is available free energy relative to its environment: exergy.

Definition 8.1 Exergy consumption

Let $B(p)$ denote the exergy consumed by a process p . At environmental temperature T_0 , irreversibility destroys exergy at the rate $T_0 \Sigma$. Thus $\Sigma > 0$ is irreversible degradation of useful free-energy resources.

9. Thermodynamically feasible operations

Definition 9.1 Budget feasibility

Fix a budget $b \in Q$. An operation f is b -feasible when $c(f) \leq b$. Write $O_{\leq b} \hookrightarrow O$ for the collection of b -feasible operations. Budgets compose by the monoidal structure $b_1 \oplus b_2$. For scalar exergy, $b_1 \oplus b_2 = b_1 + b_2$.

10. Budget-indexed operadic composition

If $f : (y_1, \dots, y_n) \rightarrow z$ costs at most b_0 and each g_i costs at most b_i , then $f \circ (g_1, \dots, g_n)$ has a realization costing at most $b_0 + \sum_i b_i$.

Theorem 10.1 Operadic cost inequality

$$c(f \circ (g_1, \dots, g_n)) \leq c(f) + \sum_i c(g_i).$$

Proof.

Choose realizers arbitrarily close to the respective infima and substitute them operadically. Costs of the resulting process are additive or lax-additive. Taking infima yields the inequality.

The process (∞) -operad therefore carries a cost enrichment.

11. The thermodynamic production doctrine

Definition 11.1 Feasible production

Let β_X be the admissible environmental exergy supply of X . Define $\Phi_X^{\beta_X}(A)$, also written $\Phi_X^{\text{th}}(A)$, to be the suboperad of operations which can be produced using operations in A , which satisfy the physical constraints of X , and which possess realizers compatible with the budget β_X . The recursively generated operad is $G_X^{\text{th}} := \mu\Phi_X^{\text{th}}$.

12. Thermodynamic organizational closure

Definition 12.1 Feasible closure

The system X is thermodynamically organizationally closed when $R_X \subseteq G_X^{\text{th}}$.

This is strictly stronger than $R_X \subseteq G_X$. Formal generability does not imply energetic feasibility. A system may satisfy the first inclusion and fail the second. It then has an abstract self-production network that it cannot energetically sustain.

13. State objects

Let $S_X \in E$ be the state object of X . A point of S_X packages the state variables that matter: concentrations, temperature, chemical potentials, the boundary ∂X , and whatever else the model treats as state.

14. The viability subobject

Definition 14.1 Viability

A viability condition is a subobject $V_X \hookrightarrow S_X$. Internally, $V_X(s)$ means that the state s is compatible with continued existence of X as a system of the specified organizational type. Viability may constrain membrane integrity, temperature, chemical potential, ATP/ADP ratio, pH, structural damage, error rates, voltage, or other variables.

Homeostasis is a viability problem. Critical variables are held in restricted ranges despite disturbances, and holding them there costs continuing free energy.

15. Admissible controls

For $s \in S_X$, let $\text{Adm}_X(s)$ be the space of thermodynamically admissible endogenous process-families available at s . Each $u \in \text{Adm}_X(s)$ produces a successor $T_X(s, u) \in S_X$. The deterministic notation may be replaced by a stochastic kernel. The lattice argument is unchanged.

16. The viability predecessor

For $A \hookrightarrow S_X$, declare $s \in \text{Pre}_X(A)$ if and only if some admissible endogenous process u satisfies $T_X(s, u) \in A$. Define $\Psi_X(A) := V_X \cap \text{Pre}_X(A)$.

A state lies in $\Psi_X(A)$ when it is viable and can be driven, admissibly, into A .

17. Monotonicity

Lemma 17.1 The viability operator is monotone

If $A \leq B$, then $\Psi_X(A) \leq \Psi_X(B)$.

Proof.

A process taking s into A also takes s into B whenever $A \leq B$. Thus $\text{Pre}_X(A) \leq \text{Pre}_X(B)$. Intersection with V_X preserves the inequality.

18. The viability kernel

The subobject lattice $\text{Sub}(S_X)$ is complete. A monotone endomap has a greatest fixed point.

Definition 18.1 Viability kernel

Set $V_X := \nu \Psi_X$. This is the viability kernel of X . A state lies in V_X precisely when some indefinitely extendable sequence of thermodynamically admissible processes keeps the system inside V_X .

Theorem 18.2 Fixed-point equation

$$V_X = V_X \cap \text{Pre}_X(V_X).$$

Proof.

Unfold $\Psi_X(V_X) = V_X$.

19. Two fixed points

The theory now contains two recursive constructions of opposite variance. The least fixed point $\mu\Phi_X^{\text{th}}$ asks which organizational operations the system can recursively generate. The greatest fixed point $\nu\Psi_X$ asks from which states the system can continue indefinitely without leaving its viability region.

20. Thermodynamically sustained autopoiesis

Definition 20.1 Thermodynamic autopoiesis

The system X is thermodynamically autopoietic, written $\text{TA}(X)$, when

$$R_X \subseteq \mu\Phi_X^{\text{th}} \quad \text{and} \quad \nu\Psi_X \neq 0.$$

The inequality 0 denotes the initial subobject. The kernel is required to be inhabited.

21. Why both fixed points are required

A self-copying chemical process might meet the operadic condition and immediately exhaust its free energy. Then $R_X \subseteq \mu\Phi_X$ while $\nu\Psi_X = 0$. A passive crystal might persist for a very long time inside a stable viability region and possess no recursively self-producing operad. Then $\nu\Psi_X \neq 0$ does not yield $R_X \subseteq \mu\Phi_X$.

Self-production together with indefinitely viable maintenance is strictly stronger than either condition alone.

22. Living systems are not thermodynamically closed

A living cell generally satisfies $\dot{E}_{\text{in}} > 0$, $J_{\text{matter}} \neq 0$, and $\dot{\Sigma} > 0$. The relevant condition is not thermodynamic closure.

Organizational closure is sustained by thermodynamic openness.

Nonequilibrium thermodynamics treats cellular processes as dissipative systems held away from equilibrium by sustained chemical or energetic driving. There is no contradiction between the two fixed points and that fact. The driving is what lets the least fixed point stay feasible and the greatest fixed point stay inhabited.

23. Maintenance cost

Define the minimal average maintenance power of X by

$$m_X := \inf_{\pi} \sup_{T \rightarrow \infty} \frac{1}{T} \int_0^T B_{\pi}(t) dt,$$

where π ranges over viable policies whose trajectories remain in V_X .

Definition 23.1 Strictly positive maintenance

The system X has strictly positive maintenance cost when $m_X > 0$.

For nonequilibrium target states this is the expected situation. Maintaining a prescribed distribution away from equilibrium requires continuing work and continuing dissipation. Explicit lower bounds of this form are known for Markov systems.

24. An energetic obstruction

Theorem 24.1 Insufficient exergy empties the kernel

Suppose $m_X > 0$ and the asymptotic available exergy power satisfies $P_X < m_X$. Then $V_X = 0$.

Proof.

If $s_0 \in V_X$, there is an indefinitely viable policy π . By definition of m_X ,

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T B_\pi(t) dt \geq m_X.$$

The environment supplies only asymptotic power $P_X < m_X$, so the required cumulative exergy eventually exceeds the available cumulative exergy. The trajectory cannot continue indefinitely.

25. The cumulative form

If every viable trajectory requires at least $\varepsilon > 0$ of free energy on each interval of duration Δt , then after n intervals the required exergy is at least $n\varepsilon$. An indefinitely viable trajectory therefore requires infinite cumulative throughput. Perpetual biological organization does not require a finite store large enough to last forever. It requires a persistent supply.

26. Isolation cannot pay the maintenance

Corollary 26.1 Nonequilibrium maintenance

If V_X is separated from thermodynamic equilibrium and maintaining states in V_X has a strictly positive dissipation lower bound, then a finite-energy isolated realization of X cannot remain viable indefinitely. Symbolically, $V_X \cap E_{\text{eq}} = \emptyset$ and $m_X > 0$ imply that a finite isolated resource forces $V_X = 0$.

Autonomous organization is compatible with thermodynamics because the autonomy is organizational, not an energetic isolation.

27. The operadic maintenance burden

For each required operation $r \in R_X$, set $c_X(r) := \inf\{\Theta(p) \mid p \models r\}$. A generating family $R = \{r_i\}$ has a schematic burden $\sum_i \lambda_i c_X(r_i)$, where λ_i is the required execution frequency. For a continuous reaction network the same idea becomes an integral, or a flux–force pairing.

28. Flux and force

For a chemical reaction network with currents J_ρ and affinities A_ρ , the entropy-production rate has the standard form

$$\dot{\Sigma} = \frac{1}{T} \sum_\rho J_\rho A_\rho \geq 0.$$

The operad says which compositions exist. The valuation says what they cost. Rao and Esposito connect the conservation laws and the topology of an open network with the decomposition of thermodynamic forces and of entropy production.

29. Host coupling

Let V be a virus and H a host, interacting across an interface I . The coupled process operad is generated by $E_V \sqcup_I E_H$. The valuation extends to mixed virus–host trees.

30. Cost of a mixed tree

Lemma Cost of a mixed tree

Let T be a finite operadic tree with vertices v , each assigned a physical process of cost c_v . Then $c(T) \leq \sum_{v \in T} c_v$.

Proof.

Induct on the tree. The single-vertex case is immediate. At an internal vertex, apply Theorem 10.1 to the root and to the child subtrees.

31. Two ways a host can help

It may happen that $r \notin G_V^{\text{th}}$ while $r \in G_{V \star H}^{\text{th}}$. There are two different reasons. The host may supply an operation that V does not have. Or the host may supply the energetic conditions under which an otherwise inaccessible operation becomes feasible. Coupling enlarges both the feasible process algebra and the available thermodynamic budget.

32. Viral thermodynamic dependence

A virus can therefore satisfy $R_V \not\subseteq \mu\Phi_V^{\text{th}}$ while $R_V \subseteq \mu\Phi_{V \star H}^{\text{th}}$. This is stronger than bare causal dependence on a host. The host closes an operadic deficit and a thermodynamic deficit at once.

33. Coupling is not a free lunch

Theorem 33.1 No thermodynamic free lunch

Suppose host supplementation makes a previously unavailable operation r feasible, so $r \notin G_V^{\text{th}}$ and $r \in G_{V \star H}^{\text{th}}$. Every physical realization of r in $V \star H$ still satisfies $\Sigma(r) \geq 0$.

Proof.

The pushout enlarges the space of allowed compositions. It does not change the second-law clause of the valuation. Every realized composite is a process of P , and $\Sigma(p) \geq 0$ for every such process.

Supplementation redistributes an energetic burden. It does not delete it.

34. Subsystem cost is not total cost

From the virus's point of view, H is an environment supplying machinery and free energy. From the point of view of $V \star H$, those contributions are internal and must be paid for in the same thermodynamic ledger.

The cost charged to a subsystem is not the cost of the total system.

Apparent autonomy must not be obtained by pushing maintenance across an unstated boundary.

35. Industrial coupling of computers

Let C be a conventional computer and I the industrial system required for its continued physical reproduction. The pushout $E_C \sqcup_I E_I$ contains silicon purification, photolithography, chip fabrication, power generation, replacement, and assembly. The isolated computer does not realize these processes. Thus $R_C^{\text{phys}} \not\subseteq \mu\Phi_C^{\text{th}}$, while one may have $R_C^{\text{phys}} \subseteq \mu\Phi_{C \star I}^{\text{th}}$.

36. The dependency analogy, and its limit

The structural analogy is not “virus $\sim$ software.” A tighter analogy pairs the virus with its host against the computer with its industrial ecology.

The pair (V, H) resembles the pair (C, I) only as a dependency pattern.

Both exhibit a subsystem whose physical continuation depends on a much richer external process algebra. This remains an analogy under a coarse dependency functor. The full objects differ.

37. The cell, thermodynamically

Let B be a bacterium. The modeling hypothesis is $R_B \subseteq \mu\Phi_B^{\text{th}}$ and $\nu\Psi_B \neq 0$, provided the environment supplies nutrients, electron donors or acceptors, light, or some other free-energy source in an admissible way. Then $\text{TA}(B)$. The cell is not energetically self-sufficient.

The cell internally realizes the machinery that converts environmental free energy into continued self-production.

That is the stronger distinction. Essay I separated closure from substrate. Essay II separated closure from a snapshot of available operations. The present condition separates a self-maintaining converter of free energy from a device whose fabrication remains outside its own operad.

38. The thermodynamic invariant

Write

$$T(X) = (R_X, \mu\Phi_X^{\text{th}}, \nu\Psi_X, m_X, \Theta_X).$$

With material typing included,

$$T_K(X) = (\text{Supp}(X), E_X, R_X, \mu\Phi_X^{\text{th}}, \nu\Psi_X, m_X, \Theta_X).$$

39. Thermodynamic equivalence

Definition 39.1 Equivalence

An equivalence $X \simeq Y$ of thermodynamically structured systems preserves E, R, Φ^{th} , the viability subobject V , the operator Ψ , and the valuation Θ .

40. Invariance

Theorem 40.1 Equivalence preserves both fixed points

If $X \simeq Y$ thermodynamically, then $F_*(\mu\Phi_X^{\text{th}}) \simeq \mu\Phi_Y^{\text{th}}$ and $F_*(\nu\Psi_X) \simeq \nu\Psi_Y$. Consequently $\text{TA}(X)$ if and only if $\text{TA}(Y)$.

Proof.

The least-fixed-point claim is the transfinite comparison used for the unweighted production operator: equivalence conjugates Φ^{th} and preserves the colimits that build $\mu\Phi$. For the viability operator, equivalence induces an isomorphism

$F_* : \text{Sub}(S_X) \xrightarrow{\sim} \text{Sub}(S_Y)$ intertwining Ψ_X and Ψ_Y . Greatest fixed points are transported to greatest fixed points.

Preservation of R gives the equivalence of TA .

41. Thermodynamic separation

Theorem 41.1 Cell and isolated computer

Suppose $TA(B)$ for a bacterium B and $\neg TA(C)$ for an isolated computer C . Then $B \neq C$ as thermodynamically structured systems.

Proof.

Theorem 40.1.

42. The virus is a different object from the infected cell

Suppose $TA(B)$, while an isolated virus satisfies $R_V \not\subseteq \mu\Phi_V^{\text{th}}$. Then $B \neq V$, again by Theorem 40.1, since thermodynamic autopoiesis fails for V . The inclusion $R_V \subseteq \mu\Phi_{V \star H}^{\text{th}}$ does not disturb the conclusion. The systems V and $V \star H$ are different.

43. Changing the boundary changes the object

Let $X \hookrightarrow Y$ be an enlargement of the system boundary. In general $E_X \subseteq E_Y$, and the available exergy may satisfy $\beta_X \leq \beta_Y$. Under compatible production doctrines, $\mu\Phi_X^{\text{th}} \subseteq \mu\Phi_Y^{\text{th}}$. A system can fail autonomy at one boundary and satisfy it at another. This is not a paradox. The object has changed.

44. No self-sufficiency claim without a boundary

For a computer the tower of boundaries runs from the machine, to the machine plus a power grid, to the machine plus a factory, to the machine plus an industrial economy. For a virus it runs from the virion, to the virion plus a host, to the virion plus host plus a transmission ecology. For a bacterium it runs from the cell to the cell plus its nutrient environment.

Every open physical system depends on an environment. The question that distinguishes them is which organizational operations are internal to the chosen boundary.

45. Feedstock is not machinery

Suppose an organism imports glucose. The molecule is a resource object. The organism already contains the processes that carry it through transport, glycolysis, respiration, ATP production, and biosynthesis. When a computer receives a newly fabricated processor, much of the process algebra that produced the processor lies outside the computer. External dependence has a type.

There is a difference between importing feedstock and importing the organizational machinery itself.

46. Two operads, not one pile of matter

Let F_X be the operad of permitted feedstock transformations and M_X the operad of machinery-producing transformations. Autopoietic closure asks, approximately, that

$$R_X^{\text{machinery}} \subseteq \mu\Phi_X(E_X),$$

and does not ask that every material particle required by X already lie in X . Organisms do not fail autonomy by eating.

47. What a bacterium imports

A bacterium may import carbon, nitrogen, phosphate, and free energy. It uses recursively produced machinery of its own to turn those imports into proteins, lipids, nucleic acids, ribosomes, membranes, and daughter cells. The intended condition is not the absence of external inputs.

External feedstock, together with endogenous productive organization, yields continued endogenous productive organization.

48. The ordinary computer has the opposite architecture

For a conventional computer the corresponding physical modes look like external machinery plus external feedstock yielding replacement components. The computer itself performs little or none of ore extraction, silicon purification, wafer fabrication, lithography, packaging, or mechanical assembly. The machinery-producing suboperad is external. This is a stronger claim than the observation that the computer needs electricity.

49. The theory does not ban artificial life

Nothing here defines life by carbon chemistry. Suppose an artificial system A had endogenous processes for resource acquisition, refining, component fabrication, repair, assembly, and reproduction of its controller, under a sustainable energy throughput. If $R_A \subseteq \mu\Phi_A^{\text{th}}$ and $\nu\Psi_A \neq 0$, then $\text{TA}(A)$. The predicate is structural. A future machine could satisfy it. Satisfying it is a different claim from computation, learning, self-modifying software, or behavioral copying.

50. Autopoiesis does not classify up to equivalence

Even if $\text{TA}(A)$ and $\text{TA}(B)$ both hold, $A \approx B$ does not follow. The supports may differ, $\text{Supp}(A) \neq \text{Supp}(B)$. The operads may differ while sharing the same closure truth value. Thermodynamic autopoiesis is an invariant, not a complete classifier. Essay I already separated the substrate obstruction from the organizational one. Both survive the thermodynamic enrichment.

51. Viability as a modality

Let $j : \Omega \rightarrow \Omega$ be a Lawvere–Tierney topology encoding a chosen notion of viable observability, with left-exact localization $L_j : E \rightarrow E_j$. Two systems may become indistinguishable after viability localization, $L_j X \approx L_j Y$, without being equivalent in the ambient topos.

A 2026 treatment of autopoiesis proposes viability-localized self-production, with a Lawvere–Tierney topology, an explicit production and repair structure, boundary production, and viability conditions. The fixed-point construction here is complementary. Rather than only localizing at viable observations, it computes the invariant set of states that support indefinitely extendable viable dynamics.

52. Localization and the kernel together

One may ask both for $L_j(R_X) \subseteq L_j(\mu\Phi_X^{\text{th}})$ and for $\nu\Psi_X \neq 0$. The first is closure up to viability-relevant equivalence. The second is the existence of actual viable trajectories. Write $\text{VTA}(X)$ for the conjunction. It is stronger than either half.

53. A viability localization is not a physical equivalence

Suppose a computational model C reproduces every viability-relevant observable of a cell B , so $L_j C \approx L_j B$. It still does not follow that $C \approx B$. The missing inference is reflection of equivalences. Unless L_j is conservative on the relevant objects, $L_j C \approx L_j B$ is only a localized equivalence.

54. A scale of equivalence

The notions of sameness in play, from coarsest to finest, are behavioral equivalence, computational equivalence, dependency equivalence, operadic equivalence, thermodynamic-operadic equivalence, and materially typed physical equivalence. None of the reverse implications is automatic. Each step remembers structure the previous step was allowed to forget, and forgetting is exactly where conservativity fails.

55. The full invariant

Define

$$I(X) = (\text{Supp}(X), E_X, R_X, \mu\Phi_X^{\text{th}}, V_X, \nu\Psi_X, \Theta_X, m_X, \text{Her}(X), \text{Lin}(X)).$$

The tuple records substrate, process algebra, recursive generability, thermodynamic feasibility, viability, maintenance cost, heredity, and lineage.

56. Full invariance

Theorem 56.1 Equivalence preserves the full invariant

If $X \simeq Y$ as fully thermodynamically structured physical systems, then $I(X) \simeq I(Y)$.

Proof.

Each coordinate is defined from structure the equivalence preserves. Support is invariant under typed equivalence. The operads E and R are preserved by definition. Least production fixed points are preserved by conjugacy of Φ . Greatest viability fixed points are preserved by conjugacy of Ψ . The valuation and the maintenance cost are preserved by thermodynamic equivalence. Heredity and lineage are part of the structured equivalence. The tuple is therefore preserved.

57. Main separation theorem

Theorem 57.1 Thermodynamic-operadic separation

Let B , V , and C be a bacterium, a virus, and a conventional computer. Assume

$$R_B \subseteq \mu\Phi_B^{\text{th}}, \quad \nu\Psi_B \neq 0,$$

while $R_V \not\subseteq \mu\Phi_V^{\text{th}}$ but $R_V \subseteq \mu\Phi_V^{\text{th}} \star_H$ for some host H , and $R_C^{\text{phys}} \not\subseteq \mu\Phi_C^{\text{th}}$ but $R_C^{\text{phys}} \subseteq \mu\Phi_C^{\text{th}} \star_I$ for some industrial environment I . Then $B \neq V$ and $B \neq C$ as thermodynamically structured systems. If furthermore $I(V) \neq I(C)$, then $V \neq C$.

Proof.

The first two statements are invariance of thermodynamic autopoiesis, Theorem 40.1. For the third, an equivalence would identify the full invariant by Theorem 56.1.

58. The diagram

The theory has three layers. Exergy β_X enters from the environment. Endogenous operations generate a feasible closure, which is asked to contain the requirements. States evolve by Ψ_X , and autonomy additionally requires the viability kernel to be inhabited.

$$\begin{array}{c}
\beta_X \\
\downarrow \\
E_X \longrightarrow \mu\Phi_X^{\text{th}} \longleftarrow R_X \\
\downarrow \\
S_X \\
\downarrow \Psi_X \\
\nu\Psi_X \subseteq V_X
\end{array}$$

The top asks what can be produced. The horizontal arrows ask what is required. The bottom asks whether production can sustain a viable trajectory.

59. The defining equations

Thermodynamically sustained autopoiesis is $R_X \subseteq G_X^{\text{th}}$ and $V_X \neq 0$. A host-dependent replicator fails the first inclusion for G_X^{th} and satisfies it for $G_X^{\text{th}} \star_H$. An energetically impossible system has $V_X = 0$, whatever its formal operad says.

- Autopoietic: $R_X \subseteq G_X^{\text{th}}$ and $V_X \neq 0$
- Host-dependent: $R_X \not\subseteq G_X^{\text{th}}$, but $R_X \subseteq G_X^{\text{th}} \star_H$
- Energetically impossible: $V_X = 0$

60. Conclusion

The resulting distinction is stronger than substrate alone and stronger than computation alone. The structures that matter are material support, the endogenous process operad, recursive productive closure, thermodynamic feasibility, viability, and the typing of environmental coupling.

The biological pattern is not “information processor.” It is closer to the following chain. Environmental matter and free energy enter a recursively self-generated process algebra. That algebra regenerates itself. The trajectory remains in a viable nonequilibrium region. In symbols, $R_X \subseteq \mu\Phi_X^{\text{th}}$ and $\nu\Psi_X \neq 0$. The thermodynamic cost of maintaining the condition cannot in general vanish: $m_X > 0$.

The organization is causally closed in the relevant organizational sense, and thermodynamically open. Thermodynamic openness is what lets organizational closure persist.

For the cell, a large part of the machinery required to construct, repair, regulate, and reproduce the system lies in $\mu\Phi_B^{\text{th}}$. For an ordinary computer, the crucial fabrication and maintenance operations lie in $\mu\Phi_C^{\text{th}} \star_I$ only after the industrial environment has been adjoined.

Internally sustained organization is not the same thing as an externally sustained artifact.

A future artificial system could cross the boundary drawn here. Demonstrating that would require more than computation, learning, self-modifying software, or behavioral self-replication. One would have to establish $R_A \subseteq \mu\Phi_A^{\text{th}}$ and $\nu\Psi_A \neq 0$, under an explicit physical boundary and a complete thermodynamic accounting. Only then would the separation proved in this edition fail to distinguish that system from an organism.

References

Jacob Lurie, **Higher Algebra**. Symmetric monoidal (∞) -categories and (∞) -operads.

John C. Baez and Blake S. Pollard, “A compositional framework for reaction networks.” **Reviews in Mathematical Physics** 29 (2017), 1750028.

John C. Baez, Brendan Fong, and Blake S. Pollard, “A compositional framework for Markov processes.” **Journal of Mathematical Physics** 57 (2016), 033301.

Riccardo Rao and Massimiliano Esposito, “Nonequilibrium thermodynamics of chemical reaction networks: wisdom from stochastic thermodynamics.” **Physical Review X** 6 (2016), 041064. Nonequilibrium free energies, chemical driving, and entropy production for open networks.

Riccardo Rao and Massimiliano Esposito, “Conservation laws shape dissipation.” **New Journal of Physics** 20 (2018), 023007.

Jordan M. Horowitz, Kevin Zhou, and Jeremy L. England, “Minimum energetic cost to maintain a target nonequilibrium state.” **Physical Review E** 95 (2017), 042102. The maintenance-cost invariant m_X .

F. W. Lawvere, “Metric spaces, generalized logic, and closed categories.” **Rendiconti del Seminario Matematico e Fisico di Milano** 43 (1973), 135–166. Enrichment over $([0, \infty], \geq, +)$, and the inequality $c(g \circ f) \leq c(f) + c(g)$.

The 2026 topos-theoretic treatments of autopoiesis cited in the essay — viability localization, production and repair, and boundary maintenance — are complementary to the two-fixed-point criterion used here, not substitutes for it.